Business Mathematics and Statistics

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Chartered Association of Certified Accountants (ACCA)
Chartered Institute of Management Accountants (CIMA)
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Each question used is cross referenced to the appropriate Institute or Association.

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Contents

Preface ..... **v**

1 Introduction to Business Mathematics and Statistics 1

Part 1 Data and their presentation **5**

2 Sampling and Data Collection 6
3 Data and their Accuracy 24
4 Frequency Distributions and Charts 38
5 General Charts and Graphs 63
Examination questions 90

Part 2 Statistical measures **95**

6 Arithmetic Mean 96
7 Median 107
8 Mode and Other Measures of Location 117
9 Measures of Dispersion and Skewness 129
10 Standard Deviation 136
11 Quantiles and the Quartile Deviation 148
Examination example and questions 159

Part 3 Regression and correlation **165**

12 Linear Functions and Graphs 166
13 Regression Techniques 173
14 Correlation Techniques 191
Examination examples and questions 207

Part 4 Time series analysis **213**

15 Time Series Model 214
16 Time Series Trend 219
17 Seasonal Variation and Forecasting 229
Examination example and questions 242

Part 5 Index numbers **247**

18 Index Relatives 248
19 Composite Index Numbers 259
20 Special Published Indices 272
Examination questions 281

Part 6 Compounding, discounting and annuities **285**

21 Interest and Depreciation 286
22 Present Value and Investment Appraisal 302
23 Annuities 318
Examination examples and questions 330

Part 7 Business equations and graphs **337**
24 Functions and Graphs 338
25 Linear Equations 351
26 Quadratic and Cubic Equations 364
27 Differentiation and Integration 374
28 Cost, Revenue and Profit Functions 385
Examination examples and questions 395

Part 8 Probability **403**
29 Set Theory and Enumeration 404
30 Introduction to Probability 419
31 Conditional Probability and Expectation 436
Examination examples and questions 449

Part 9 Further probability **455**
32 Combinations and Permutations 456
33 Binomial and Poisson Distributions 462
34 Normal Distribution 473
Examination example and questions 490

Part 10 Specialised business applications **495**
35 Linear Inequalities 496
36 Matrices 508
37 Inventory Control 526
38 Network Planning and Analysis 543
Examination example and questions 555

Answers to student exercises **562**

Answers to examination questions **581**

Appendices **650**
1 Compounding and Discounting Tables 650
2 Random Sampling Numbers 654
3 Exponential Tables. Values of e^{-m} 655
4 Standard Normal Distribution Tables 657

Index **659**

Preface

1. Aims of the book

The general aim of the book is to give a thorough grounding in basic Mathematical and Statistical techniques to students of Business and Professional studies. No prior knowledge of the subject area is assumed.

2. Courses covered

a) The book is intended to support the courses of the following professional bodies:
   Chartered Association of Certified Accountants
   Chartered Institute of Management Accountants
   Institute of Chartered Secretaries and Administrators
b) The courses of the following bodies which will be supported by the book to a large extent:
   Chartered Institute of Insurance
   Business and Technical Education Council (National level)
   Association of Accounting Technicians
c) The book is also meant to cater for the students of any other courses who require a practical foundation of Mathematical and Statistical techniques used in Business, Commerce and Industry.

3. Format of the book

The book has been written in a standardised format as follows:

a) There are TEN separate parts which contain standard examination testing areas.
b) Numbered chapters split up the parts into smaller, identifiable segments, each of which have their own Summaries and Points to Note.
c) Numbered sections split the chapters up into smaller logical elements involving descriptions, definitions, formulae or examples.

At the end of each chapter, there is a Student Self Review section which contains questions that are meant to test general concepts, and a Student Exercise section which concentrates on the more practical numerical aspects covered in the chapter.

At the end of each part, there is

a) a separate section containing examination examples with worked solutions and
b) examination questions from various bodies. Worked solutions to these questions are given at the end of the book.

4. How to use the book

Chapters in the book should be studied in the order that they occur.

After studying each section in a chapter, the Summaries and Points to Note should be checked through. The Student Self Review Questions, which are cross-referenced to appropriate sections, should first be attempted unaided, before checking the answers with the text. Finally the Student Exercises should be worked through and the answers obtained checked with those given at the end of the book.

After completing a particular part of the book, the relevant section of the examination questions (at the end of the book) should be attempted. These questions should be considered as an integral part of the book, all the subject matter included having been covered in previous chapters and parts. Always make some attempt at the questions before reading the solution.

5. The use of calculators

Examining bodies permit electronic calculators to be used in examinations. It is therefore essential that students equip themselves with a calculator from the beginning of the course.

Essential facilities that the calculator should include are:

a) a square root function, and

b) an accumulating memory.

Very desirable extra facilities are:

c) a power function (labelled 'x^y'),

d) a logarithm function (labelled 'log x'), and

e) an exponential function (labelled 'e^x').

Some examining bodies exclude the use (during examinations) of programmable calculators and/or calculators that provide specific statistical functions such as the mean or the standard deviation. Students are thus urged to check on this point before they purchase a calculator. Where relevant, this book includes sections which describe techniques for using calculators to their best effect.

Andre Francis, 2004

1 Introduction to business mathematics and statistics

1. Introduction

This chapter serves as an introduction to the whole book. It describes the main areas covered under the heading 'Business Mathematics and Statistics' and introduces the idea of a statistical investigation.

2. Differences in terminology

The title of this book is Business Mathematics and Statistics. However, many other terms are used in business and by Professional bodies to describe the same subject matter. For example, Quantitative Methods, Quantitative Techniques and Numerical Analysis.

3. Business mathematics and statistics

A particular problem for management is that most decisions need to be taken in the light of incomplete information. That is, not everything will be known about current business processes and very little (if anything) will be known about future situations. The techniques described in 'Business Mathematics and Statistics' enable structures to be built up which help management to alleviate this problem. The main areas included in the book are: (a) Statistical Method; (b) Management Mathematics; and (c) Probability.

These areas are described briefly in the following sections.

4. Statistical method

Statistical method can be described as:

a) the selection, collection and organisation of basic facts into meaningful data, and then

b) the summarizing, presentation and analysis of data into useful information.

The gap between facts as they are recorded (anywhere in the business environment) and information which is useful to management is usually a large one. (a) and (b) above describe the processes that enable this gap to be bridged. For example, management would find percentage defect rates of the fleets of lorries in each branch more useful than the daily tachometer readings of individual vehicles. That is, management generally require summarized values which represent large areas under their control, rather than detailed figures describing individual instances which may be untypical.

Note that the word 'Statistics' can be used in two senses. It is often used to describe the topic of Statistical Method and is also commonly used to describe values which summarize data, such as percentages or averages.

5. Management mathematics

The two areas covered in this book which can be described as Management Mathematics are described as follows:

a) *The understanding and evaluation of the finances involved in business investments*. This involves considering interest, depreciation, the worth of future cash flows (present value), various ways of repaying loans and comparing the value of competing investment projects;
b) *Describing and evaluating physical production processes in quantitative terms*. Techniques associated with this area enable the determination of the level of production and prices that will minimise costs or maximise the revenue and profits of production processes;

Involved in both of the above are the manipulation of algebraic expressions, graph drawing and equation solving.

6. Probability

Probability can be thought of as the ability to attach limits to areas of uncertainty. For example, company profit for next year is an area of uncertainty, since there will never be the type of information available that will enable management to forecast its value precisely. What can be done however, given the likely state of the market and a range of production capacity, is to calculate the limits within which profit is likely to lie. Thus calculations can be performed which enable statements such as 'there is a 95% chance that company profit next year will lie between £242,000 and £296,000' to be made.

7. Statistical investigations

Management decisions are based on numerous pieces of information obtained from many different sources. They may have used one, some or all of the techniques which have been described as Statistical Method, Management Mathematics or Probability. What the decisions will all have in common however is that they are the final product of a general structure (or set of processes) known as an *investigation* or *survey*. Some significant factors are listed as follows.

a) Investigations can be fairly trivial affairs, such as looking at today's orders to see which are to be charged to credit or cash. Others can be major undertakings, involving hundreds of staff and a great deal of expense over a number of years, such as the United Kingdom Population Census (carried out every ten years).
b) Investigations can be carried out in isolation or in conjunction with others. For example, the calculation of the official monthly Retail Price Index involves a major (ongoing) investigation which includes using the results of the Family Expenditure Survey (which is used also for other purposes). However, the information needed for first line management to control the settings of machines on a production line might depend only on sampling output at regular intervals.
c) Investigations can be regular (routine or ongoing) or 'one-off'. For example, the preparation of a company's trial balance as against a special investigation to examine the calculation of stock re-order levels.
d) Investigations are carried out on populations. A *population* is the entirety of people or items (technically known as *members*) being considered. Thus if a company wanted information on the time taken to complete jobs, the population would consist of all jobs started in the last calendar year say. Sometimes complete populations are investigated, but often only representative sections of

the population, or *samples*, are surveyed due to time, manpower and resource restrictions.

8. Stages in an investigation

However small or large an investigation is, there are certain landmarks or identifiable stages, through which it should normally pass. These are listed as follows.

a) *Definition of target population and objectives of the survey*. Who? (e.g. does the term 'workers' include temporary part-timers?) Why? (Answering this correctly will ensure that unnecessary questions are not asked and essential questions are asked.)
b) *Choice of method of data collection*. Sometimes a survey will dictate which method is used and in other cases there will be a choice. A list of the most common methods of data collection is given in the following chapter.
c) *Design of questionnaire* or the specification of other criteria for data measurement.
d) *Implementation of a pilot (or trial) survey*. A pilot survey is a small 'pre-survey' carried out in order to check the method of data collection and ensure that questions to be asked are of the right kind. Pilot surveys are normally carried out in connection with larger investigations, where considerable expenditure is involved.
e) *Selection of population members to be investigated*. If the whole (target) population is not being investigated, then a method of sampling from it must be chosen. Various sampling methods are covered fully in the following chapter.
f) *Organisation of manpower and resources to collect the data*. Depending on the size of the investigation, there are many factors to be considered. These might include: training of interviewers, transport and accommodation arrangements, organisation of local reporting bases, procedures for non-responses and limited checking of replies.
g) *Copying, collation and other organisation* of the collected data.
h) *Analyses* of data (with which much of the book is concerned).
i) *Presentation* of analyses and preparation of reports.

9. Summary

a) The subject matter of this book, Business Mathematics and Statistics, is sometimes described as Quantitative Methods, Quantitative Techniques or Numerical Analysis.
b) The subject matter attempts to alleviate the problem of incomplete information for management under the three broad headings: Statistical Method, Management Mathematics and Probability.
c) The gap between facts as they are recorded and the provision of useful information for management is bridged by Statistical Method. This covers:
    i. the selection, collection and organisation of basic facts into meaningful data, and
    ii. the summarizing, presentation and analysis of data into useful information.

d) The extent of the Management Mathematics that is covered in this book is concerned with:
   i. the understanding and evaluation of the finances involved in business investments, and
   ii. describing and evaluating physical production processes in quantitative terms.
e) Probability can be thought of as the ability to attach limits to areas of uncertainty.
f) Statistical investigations can be considered as the logical structure through which information is provided for management. They can be: trivial or major; carried out in isolation or in conjunction with other investigations; regular or 'one-off'. Investigations are carried out on populations, which can be described as the entirety of people or items under consideration.
g) The stages in an investigation could be some or all of the following, depending on their size and scope.
   i. Definition of target population and survey objectives.
   ii. Choice of method of data collection.
   iii. Design of questionnaire or the specification of other criteria for data measurement.
   iv. Implementation of a pilot survey.
   v. Selection of population members to be investigated.
   vi. Organisation of manpower and resources to collect the data.
   vii. Copying, collation and other organisation of the collected data.
   viii. Analyses of data.
   ix. Presentation of analyses and preparation of reports.

10. Student self review questions

1. What does management use Business Mathematics and Statistics for? [3]
2. What is Statistical Method and what purpose does it serve? [4]
3. Describe the two main areas covered under the heading of Management Mathematics. [5]
4. What is Probability? [6]
5. What is the particular significance of a statistical investigation to management information? [7]
6. What is meant by the term 'population'? [7]
7. List the stages of a statistical investigation. [8]

Part 1 Data and their presentation

This part of the book deals with the origins, organisation and presentation of statistical data.

Chapter 2 describes methods of selecting data items for investigation (using censuses and samples) and the various ways in which data can be collected.

Data are classified in chapter 3 and some aspects of their accuracy, including rounding, is discussed.

Chapter 4 covers various forms of frequency distributions, which are the main method of organising numerical data into a form which is convenient for either graphical presentation or analysis. Charts used to display frequency distributions include histograms and Lorenz curves.

Chapter 5 describes the many types of charts and graphs that are used to describe non-numeric data and data described over time. These include several types of bar charts, pie charts, and line diagrams.

2 Sampling and data collection

1. Introduction

This chapter is concerned with the various methods employed in choosing the subjects for an investigation and the different ways that exist for collecting data. Primary data sources (censuses and samples) are described in depth and include:

a) advantages and disadvantages in their use, and
b) data collection techniques.

Secondary data sources, mainly official publications, are covered later in the chapter.

2. Primary and secondary data

a) *Primary data* is the name given to data that are used for the specific purpose for which they were collected. They will contain no unknown quantities in respect of method of collection, accuracy of measurements or which members of the population were investigated. Sources of primary data are either censuses or samples and both of these are described in the following sections.
b) *Secondary data* is the name given to data that are being used for some purpose other than that for which they were originally collected. Summaries and analyses of such data are sometimes referred to as *secondary statistics*. The main sources of secondary data are described in later sections of the chapter.

Statistical investigations can use either primary data, secondary data or a combination of the two. An example of the latter follows. Suppose that a national company is planning to introduce a new range of products. It might refer to secondary data on rail and road transport, areas of relevant skilled labour and information on the production and distribution of similar goods from tables provided by the Government Statistical Service to site their new factory. The company might also have carried out a survey to produce their own primary data on prospective customer attitudes and the availability of distribution through wholesalers.

3. Censuses

A *census* is the name given to a survey which examines *every member* of a population.

a) A firm might take a census of all its employees to find out their opinions on the possible introduction of a new incentive scheme.
b) The Government Statistical Service carries out many official censuses. Some of them are described as follows.
   i. A *Population Census* is taken every ten years, obtaining information such as age, sex, relationship to head of household, occupation, hours of work, education, use of a car for travel to work, number of rooms in place of dwelling etc for the whole population of the United Kingdom.
   ii. A *Census of Distribution* is taken every five years, covering virtually all retail establishments and some wholesalers. It obtains information on numbers of employees, type of goods sold, turnover and classification etc.
   iii. A *Census of Production* is taken every five years, covering manufacturing industries, mines and quarries, building trades and public utility produc-

tion services. The information obtained and analysed includes distribution of labour, allocation of capital resources, stocks of raw materials and finished goods and expenditure on plant and machinery.

A census has the obvious advantages of completeness and being accepted as representative, but of course must be paid for in terms of manpower, time and resources. The three government censuses described above involve a great deal of organisation, with some staff needed permanently to answer queries on the census form, check and correct errors and omissions and extensively analyse and print the information collected. Forms can take up to a year to be returned with a further gap of up to two years before the complete results are published.

4. Samples

In practice, most of the information obtained by organisations about any population will come from examining a small, representative subset of the population. This is called a *sample*. For example:

i. a company might examine one in every twenty of their invoices for a month to determine the average amount of a customer order;
ii. a newspaper might commission a research company to ask 1000 potential voters their opinions on a forthcoming election.

The information gathered from a sample (i.e. measurements, facts and/or opinions) will normally give a good indication of the measurements, facts and/or opinions of the population from which it is drawn. The *advantages* of sampling are usually smaller costs, time and resources. A general *disadvantage* is a natural resistance by the layman in accepting the results as representative. Other disadvantages depend on the particular method of sampling used and are specified in later sections, when each sampling method is described in turn.

5. Bias

Bias can be defined as the tendency of a pattern of errors to influence data in an unrepresentative way. The errors involved in the results of investigations that have been subject to bias are known as systematic errors.

The main types of bias are now described.

a) *Selection bias*. This can occur if a sample is not truly representative of the population. Note that censuses cannot be subject to this type of bias. For example, sampling the output from a particular machine on a particular day may not adequately represent the nature and quality of the goods that customers receive. Factors that could be involved are: there may be other machines that perform better or worse; this machine might be manned by more or less experienced operators; this day's production may be under more or less pressure than another day's.
b) *Structure and wording bia*s. This could be obtained from badly worded questions.

   For example, technical words might not be understood or some questions may be ambiguous.

c) *Interviewer bias*. If the subjects of an investigation are personally interviewed, the interviewer might project biased opinions or an attitude that might not gain the full cooperation of the subjects.

d) *Recording bias*. This could result from badly recorded answers or clerical errors made by an untrained workforce.

6. Sampling frames

Certain sampling methods require each member of the population under consideration to be known and identifiable. The structure which supports this identification is called a *sampling frame*. Some sampling methods require a sampling frame only as a listing of the population; other methods need certain characteristics of each member also to be known. Sampling frames can come in all shapes and sizes. For example:

i. A firm's customers can be identified from company records.
ii. Employees can be identified from personnel records.
iii. A sampling frame for the students at a college would be their enrolment forms.
iv. The relevant telephone book would form a sampling frame of people who have telephones in a certain area.
v. Stock items can be identified from an inventory file.

Note however that there are many populations that might need to be investigated for which no sampling frame exists. For example, a supermarket's customers, items coming off a production line or the potential users of a new product. Sampling techniques are often chosen on the basis of whether or not a sampling frame exists.

7. Sampling techniques

The sampling techniques most commonly used in business and commerce can be split into three categories.

a) *Random sampling*. This ensures that each and every member of the population under consideration has an equal chance of being selected as part of the sample. Two types of random sampling used are:
   i. Simple random sampling (see section 9), and
   ii. Stratified (random) sampling (see section 12).

b) *Quasi-random sampling*. (Quasi means 'almost' or 'nearly'.) This type of technique, while not satisfying the criterion given in a) above, is generally thought to be as representative as random sampling under certain conditions. It is used when random sampling is either not possible or too expensive to consider. Two types that are commonly used are:
   i. Systematic sampling (see section 13), and
   ii. Multi-stage sampling (see section 14).

c) *Non-random sampling*. This is used when neither of the above techniques are possible or practical. Two well-used types are:
   i. Cluster sampling (see section 15), and
   ii. Quota sampling (see section 16).

Before covering each of the above sampling methods in turn, it is necessary to describe some associated concepts and structures.

8. Random sampling numbers

The two types of random sampling, listed in section 7 above and described in sections 9 and 12 following, normally require the use of *random sampling numbers*. These consist of the ten digits from 0 to 9, generated in a random fashion (normally from a computer) and arranged in groups for reading convenience. The term 'generated in a random fashion' can be interpreted as 'the chance of any one digit occurring in any position in the table is no more or less than the chance of any other digit occurring'.

Appendix 2 shows a typical table of such numbers, blocked into groups of five digits. The table is used to ensure that any random sample taken from some sampling frame will be free from bias. The following section describes the circumstances under which the tables are used.

9. Simple random sampling

Simple random sampling, as described earlier, ensures that each member of the population has an equal chance of being chosen for the sample. It is necessary therefore to have a sampling frame which (at the least) lists all members of the target population. Examples of where this method might be used are:

a) by a large company, to sample 10% of their orders to determine their average value;
b) by an auditor, to sample 5% of a firm's invoices for completeness and compatibility with total yearly turnover;
c) by a professional association, to sample a proportion of its members to determine their views on a possible amalgamation with another association.

Each of these three would have obvious, ready-made sampling frames available.

It is generally accepted that the best method of drawing a simple random sample is by means of random sampling numbers. Example 1, which follows, demonstrates how the tables are used.

The *advantages* of this method of sampling include the selection of sample members being unbiased and the general acceptance by the layman that the method is fair.

Disadvantages of the method include:

i. the need for a population listing,
ii. the need for each chosen subject to be located and questioned (this can take time), and
iii. the chance that certain significant attributes of the population are under or over represented.

For example, if the fact that a worker is part-time is considered significant to a survey, a simple random sample might only include 25% part-time workers from a population having, say, a 30% part-time work force.

10. Example 1 (Use of *random sampling numbers*)

An auditor wishes to sample 29 invoices out of a total of 583 received in a financial year. The procedure that could be followed is listed below.

1. Each invoice would be numbered, from 001 through to 583.
2. Select a starting row or column from a table of random sampling numbers and begin reading groups of three digits sequentially. For example, using the random sampling numbers at Appendix 2, start at row 6 (beginning 34819 80011 17751 03275 ...etc). This gives the groups of three as: 348 198 001 117 751 032 ... etc.
3. Each group of three digits represents the choice of a numbered invoice for inclusion in the sample. Any number that is greater than 583 is ignored as is any repeat of a number. Using the illustration from 2. above, invoice numbers 348, 198, 001, 117, 032, etc would be accepted as part of the sample, while number 751 would be rejected as too large.
4. As many groups of three digits as necessary are considered until 29 invoices have been identified. This forms the required sample.

Notes:

a) Random sampling numbers can be generated by a computer or pre-printed tables can be obtained.
b) The number of digits to be read in groups will always depend upon how many members there are in the population. If there were 56,243 members, then digits would need to be read in fives; groups of four digits would be read if a population being sampled had 8771 members.
c) The choice of a starting row or column for reading groups of digits should be selected randomly.

11. Stratification of a population

Stratification of a population is a process which:

i. identifies certain attributes (or strata levels) that are considered significant to the investigation at hand;
ii. partitions the population accordingly into groups which each have a unique combination of these levels.

For example, if whether or not heavy goods vehicles had a particular safety feature was thought important to an investigation, the population would be partitioned into the two groups 'vehicles with the feature' and 'vehicles without the feature'. On the other hand, if whether an employee was employed full or part-time, together with their sex, was felt to be significant to their attitudes to possible changes in working routines, the population would be partitioned into the four groups: male/full-time; female/full-time; male/part-time and female/part-time.

Populations that are stratified in this way are sometimes referred to as *heterogeneous*, meaning that they are composed of diverse elements or attributes that are considered significant.

12. Stratified sampling

Stratified random sampling extends the idea of simple random sampling to ensure that a heterogeneous population has its defined strata levels taken account of in the sample. For example, if 10% of all heavy goods vehicles have a certain safety feature, and this is considered significant to the investigation in hand, then 10% of a sample of such vehicles must have the safety feature.

The general procedure for taking a stratified sample is:

a) Stratify the population, defining a number of separate partitions.
b) Calculate the proportion of the population lying in each partition.
c) Split the total sample size up into the above proportions.
d) Take a separate sample (normally simple random) from each partition, using the sample sizes as defined in (c).
e) Combine the results to obtain the required stratified sample.

Stratification of a population can be as simple or complicated as the situation demands. Some surveys might warrant that a population be split into many strata. A major investigation into car safety could identify the following significant factors having some bearing on safety: saloon and estate cars; radial and cross-ply tyres; two and four-door models; rear passenger safety belts (or not). The sampling frame in this case would have to be split into sixteen separate partitions in order to take account of all the combinations possible from (i) to (iv) above (for example, saloon/radial/2-door/belts and saloon/radial/2-door/no belts are just two of the partitions).

Advantages of this method of sampling include the fact that the sample itself (as well as the method of selection) is free from bias, since it takes into account significant strata levels (attributes) of a population considered important to the investigation.

Disadvantages of stratified sampling include:

i. an extensive sampling frame is necessary;
ii. strata levels of importance can only be selected subjectively;
iii. increased costs due to the extra time and manpower necessary for the organisation and implementation of the sample.

13. Systematic sampling

Systematic sampling is a method of sampling that can be used where the population is listed (such as invoice values or the fleet of company vehicles) or some of it is physically in evidence (such as a row of houses, items coming off a production line or customers leaving a supermarket). The technique is to choose a random starting place and then systematically sample every 40th (or 12th or 165th) item in the population, the number (40, say) having been chosen based on the size of sample required. For example, if a 2% sample was needed from a population, every 50th item would be selected, after having started at some random point.

This is because 2% = 2 in 100 = 1 in 50.

Systematic sampling is particularly useful for populations that (with respect to the investigation to hand) are of the same kind or are uniform. These are referred to as *homogeneous* populations. For example, the invoices of a company for one financial year would be considered as a homogeneous population by an auditor, if their value or relationship to type of goods ordered was of no consequence to the investigation. Thus, a systematic sample could be used.

Care must be taken however, when using this method of sampling, that no set of items in the population recur at set intervals. For example, if four machines are producing identical products at the same rate and these are being passed to a single conveyer, it could happen that the products form natural sets of four (one from each machine). A systematic sample, examining every n-th item (where n is a factor of 4), might well be selecting products from the same machine and therefore be biased.

Advantages of this method include:

i. ease of use;
ii. the fact that it can be used where no sampling frame exists (but items are physically in evidence).

The main *disadvantage* of systematic sampling is that bias can occur if recurring sets in the population are possible.

This method of sampling is not truly random, since (once a random starting point has been selected) all subjects are pre-determined. Hence the use of the term 'quasi-random' to describe the technique.

14. Multi-stage sampling

Where a population is spread over a relatively wide geographical area, random sampling will almost certainly entail travelling to all parts of the area and thus could be prohibitively expensive. *Multi-stage sampling*, which is intended to overcome this particular problem, involves the following.

a) Splitting the area up into a number of regions;
b) Randomly selecting a small number of the regions;
c) Confining sub-samples to these regions alone, with the size of each sub-sample proportional to the size of the area. For example, the United Kingdom could be split up into counties or a large city could be split up into postal districts;
d) The above procedure can be repeated for sub-regions within regions... and so on.

Once the final regions (or sub-regions etc) have been selected, the final sampling technique could be (simple or stratified) random or systematic, depending on the existence or otherwise of a sampling frame.

The main *advantage* of this method is that less time and manpower is needed and thus it is cheaper than random sampling.

Disadvantages of multi-stage sampling include:

i. possible bias if a very small number of regions is selected;
ii. the method is not truly random, since, once particular regions for sampling have been selected, no member of the population in any other region can be selected.

15. Cluster sampling

Cluster sampling is a non-random sampling method which can be employed where no sampling frame exists, and, often, for a population which is distributed over some geographical area. The technique involves selecting one or more geographical areas and sampling *all* the members of the target population that can be identified.

For example, suppose a survey was needed of companies in South Wales who use a computerized payroll. First, three or four small areas would be chosen (perhaps two of these based in city centres and one or two more in outlying areas). Each company, in each area, might then be phoned, to identify which of them have computerized systems. The survey itself could then be carried out.

The *advantages* of cluster sampling include:

i it is a good alternative to multi-stage sampling where no sampling frame exists;

ii. it is generally cheaper than other methods since little organisation or structure is needed in the selection of subjects.

The main *disadvantage* of the method is the fact that sampling is not random and thus selection bias could be significant. (Non-response is not normally considered to be a particular problem.)

16. Quota sampling

A sampling technique much favoured in market research is *quota sampling*. The method uses a team of interviewers, each with a set number (quota) of subjects to interview. Normally the population is stratified in some way and the interviewer's quota will reflect this. This method places a lot of responsibility onto interviewers since the selection of subjects (and there could be many strata involved) is left to them entirely. Ideally they should be well trained and have a responsible, professional attitude.

The *advantages* of quota sampling include:

i. stratification of the population is usual (although not essential);

ii. no non-response;

iii. low cost and convenience.

The main *disadvantages* of this method are:

i. sampling is non-random and thus selection bias could be significant;

ii. severe interviewer bias can be introduced into the survey by inexperienced or untrained interviewers, since all the data collection and recording rests with them.

17. Precision

Clearly the best way of obtaining information about a population is to take a census. This will ensure (barring any bias and clerical errors) that the information obtained about the population is accurate. However, sampling is a fact of life and the information about a population that is derived from a sample will inevitably be imprecise. The error involved is sometimes known as sampling error. One technique that is often used to compensate for this is to state limits of error for any sample statistics produced. Particular precision techniques are just outside the scope of this book.

18. Sample size

There is no universal formula for calculating the size of a sample. However, as a starting point, there are two facts that are well known from statistical theory and should be remembered.

1. The larger the size of sample, the more precise will be the information given about the population.
2. Above a certain size, little extra information is given by increasing the size.

All that can be deduced from the above two statements, together with some other points made in earlier sections of the chapter, is that a sample need only be large enough to be reasonably representative of the population. Some general factors involved in determining sample size are listed below.

a) *Money and time available*.

b) *Aims of the survey*. For example, for a quick market research exercise, a very small sample (perhaps just 50 or 100 subjects) might suffice. However if the opinions of the workforce were desired on a major change of working structures, a 20 or 30% sample might be in order.

c) *Degree of precision required*. The less precise the results need to be, the smaller the sample size.

   For example, to gauge an approximate market reaction to one of their new products, a firm would only need a very small sample. On the other hand, if motor vehicles were being sampled for exhaustive safety tests at a final production stage, the sample would need to be relatively large.

d) *Number of sub-samples required*. When a stratified sample needs to be taken and many sub-samples are defined, it might be necessary to take a relatively large total sample in order that some smaller groups contain significant numbers.

   For example, suppose that a small sub-group accounted for only 0.1% of the population. A total sample size as large as 10,000 would result in a sample size of only 10 (0.1%) for this sub-group, which would probably not be large enough to gain any meaningful information.

19. Methods of primary data collection

Data collection can be thought of as the means by which information is obtained from the selected subjects of an investigation. There are various data collection methods which can be employed. Sometimes a sampling technique will dictate which method is used and in other cases there will be a choice, depending on how much time and manpower (and inevitably money) is available. The following list gives the most common methods.

a) *Individual (personal) interview*.

   This method is probably the most expensive, but has the advantage of completeness and accuracy. Normally questionnaires will be used (described in more detail in the following section).

   Other factors involved are:

   i. interviewers need to be trained;
   ii. interviews need arranging;

iii. can be used to advantage for pilot surveys, since questions can be thoroughly tested;
iv. uniformity of approach if only one interviewer is used;
v. an interviewer can see or sense if a question has not been fully understood and it can be followed-up on the spot.

This form of data collection can be used in conjunction with random or quasi-random sampling.

b) *Postal questionnaire*.

This is a much cheaper method than the personal interview since manpower (one of the most expensive resources) is not used in the data collection. However, much more effort needs to be put into the design of the questionnaire, since there is often no way of telling whether or not a respondent has understood the questions or has answered them correctly (both of these are generally no problem in a personal interview).

Other factors involved are:

i. low response rates (although inducements, such as free gifts, often help);
ii. convenience and cheapness of the method when the population is scattered geographically;
iii. no prior arrangements necessary (unlike the personal interview);
iv. questionnaires sent to a company may not be filled in by the correct person.

This method can be used in conjunction with most forms of sampling.

c) *Street (informal) interview*.

This method of data collection is normally used in conjunction with quota sampling, where the interviewer is often just one of a team. Some factors involved are:

i. possible differences in interviewer approach to the respondents and the way replies are recorded;
ii. questions must be short and simple;
iii. non-response is not a problem normally, since refusals are ignored and another subject selected;
iv. convenient and cheap.

d) *Telephone interview*.

This method is sometimes used in conjunction with a systematic sample (from the telephone book). It would generally be used within a local area and is often connected with selling a product or a service (for example, insurance). It has an in-built bias if private homes are being telephoned (rather than businesses), since only those people with telephones can be contacted and interviewed. It can cause aggravation and the interviewer needs to be very skilled.

e) *Direct observation*.

This method can be used for examining items sampled from a production line, in traffic surveys or in work study. It is normally considered to be the most accurate form of data collection, but is very labour-intensive and cannot be used in many situations.

20. Questionnaire design

If a questionnaire is used in a statistical survey, its design requires careful consideration. A badly designed questionnaire can cause many administrative problems and may cause incorrect deductions to be made from statistical analyses of the results. One of the major reasons why pilot surveys are carried out is to check typical responses to questions. Some important factors in the design of questionnaires are given below.

a) The questionnaire should be as short as possible.

b) Questions should:

   i. be simple and unambiguous.

   ii. not be technical.

   iii. not involve calculations or tests of memory.

   iv. not be personal, offensive or leading.

c) As many questions as possible should have simple answer categories (so that the respondent has only to choose one). For example:

How many employees are there in your company?

Under 10	10 to 24	25 to 49	50 or over

Do you find the equipment supplied:

Very easy to use?		Fairly easy to use?	
Fairly difficult to use?		Very difficult to use?	

d) Questions should be asked in a logical order.

A useful check on the adequacy of the design of a questionnaire can be given by conducting a *pilot survey*.

21. The use of secondary data

Secondary data are generally used when:

a) the time, manpower and resources necessary for your own survey are not available (and, of course, the relevant secondary data exists in a usable form), or

b) it already exists and provides most, if not all, of the information required.

The *advantages* of using secondary data are savings in time, manpower and resources in sampling and data collection. In other words, somebody else has done the 'spade work' already.

The *disadvantages* of using secondary data can be formidable and careful examination of the source(s) of the data is essential. Problems include the following.

i. Data quality might be questionable. For example, the sample(s) used may have been too small, interviewers may not have been experienced or any questionnaires used may have been badly designed.
ii. The data collected might now be out-of-date.
iii. Geographical coverage of the survey may not coincide with what you require. For example, you might require information for Liverpool and the secondary data coverage is for the whole of Merseyside.
iv. The strata of the population covered may not be appropriate for your purposes. For example, the secondary data might be split up into male/female and full-time/part-time workers and you might consider that, for your purposes, whether part-time workers are permanent or temporary is significant.
v. Some terms used might have different meanings. Common examples of this are:
   - Wages (basic only or do they include overtime?)
   - Level of production (are rejects included?)
   - Workers (factory floor only or are office staff included?)

22. Sources of secondary data and their use

Secondary data sources fall broadly into two categories: those that are *internal* and those *external* to the organisation conducting the survey.

Some examples of *internal* secondary data sources and uses are:

a) a customer order file, originally intended for standard accounting purposes, could have its addresses and typical goods amounts used for route planning purposes;
b) using information on raw material type and price (originally collected by the purchase department to compare manufacturers) for stock control purposes;
c) information on job times and skills breakdown, originally compiled for job costing, used for organising new pay structures.

Some examples of *external* secondary data sources are:

d) the results of a survey undertaken by a credit card company, to analyse the salary and occupation of its customers, might be used by a mail order firm for advertising purposes;
e) a commercially produced car survey giving popularity ratings and buying intentions, might be used by a garage chain to estimate stock levels of various models.

Without doubt, the most important external secondary data sources are official statistics supplied by the Central Statistical Office and other government departments. These are listed and briefly described in the next section.

23. Official secondary data sources

The following list gives the major publications of the Central Statistical Office.

a) *Annual Abstract of Statistics*. This publication is regarded as the main general reference book for the United Kingdom and has been published for nearly 150 years. Its tables cover just about every aspect of economic, social and industrial life. For example: climate; population; social services; justice and crime;

education; defence; manufacturing and agricultural production; transport and communications; finance.

b) *Monthly Digest of Statistics*. A monthly abbreviated version of the Annual Abstract of Statistics. Gives the facts on such topics as: population; employment; prices; wages; social services; production and output; energy; engineering; construction; transport; retailing; finance and the weather. It has runs on monthly and quarterly figures for at least two years in most tables and annual figures for longer periods. An annual supplement gives definitions and explanatory notes for each section. An index of sources is included.

c) *Financial Statistics*. A monthly publication bringing together the key financial and monetary statistics of the United Kingdom. It is the major reference document for people and organisations concerned with government and company finance and financial markets generally. It usually contains at least 18 monthly, 12 quarterly or 5 annual figures on a wide variety of topics. These include: financial accounts for sectors of the economy; Government income and expenditure; public sector borrowing; banking statistics; money supply and domestic credit expansion; institutional investment; company finance and liquidity; exchange and interest rates. An annual explanatory handbook contains notes and definitions.

d) *Economic Trends*. Published monthly, this is a compilation of all the main economic indicators, illustrated with charts and diagrams. The first section (Latest Developments) presents the most up-to-date statistical information available during the month, together with a calendar of recent economic events. The central section shows the movements of the key economic indicators over the last five years or so. Finally there is a chart showing the movements of four composite indices over 20 years against a reference chronology of business cycles. In addition, quarterly articles on the national accounts appear in the January, April, July and October issues, and on the balance of payments in the March, June, September and December issues. Occasional articles comment on and analyse economic statistics and introduce new series, new analyses and new methodology. *Economic Trends* also publishes the release dates of forthcoming important statistics. An annual supplement gives a source for very long runs, up to 35 years in some cases, of key economic indicators. The longer runs are annual figures, but quarterly figures for up to 25 years or more are provided.

e) *Regional Trends*. An annual publication, with many tables, maps and charts, it presents a wide range of government statistics on the various regions of the United Kingdom. The data covers many social, demographic and economic topics. These include: population, housing, health, law enforcement, education and employment, to show how the regions of the United Kingdom are developing and changing.

f) *United Kingdom National Accounts (The Blue Book)*. Published annually, this is the essential data source for those concerned with macro-economic policies and studies. The principal publication for national accounts statistics, it provides detailed estimates of national product, income and expenditure. It covers industry, input and output, the personal sector, companies, public corporations, central and local government, capital formation and national accounts. Tables of statistical information, generally extending over eleven years, are supported by definitions and detailed notes. It is a valuable indicator of how the nation makes and spends its money.

g) *United Kingdom Balance of Payments (The Pink Book)*. This annual publication is the basic reference book of balance of payments statistics, presenting all the statistical information (both current and for the preceding ten years) needed by those who seek to assess United Kingdom trends in relation to those of the rest of the world.

h) *Social Trends*. One of the most popular and colourful annual publications, it has (for over ten years) provided an insight into the changing patterns of life in Britain. The chapters provide accurate analyses and breakdowns of statistical information on population, households and families, education and employment, income and wealth, resources and expenditure, health and social services and many other aspects of British life and work.

i) *Guide to Official Statistics*. A periodically produced reference book for all users of statistics. It indicates what statistics have been compiled for a wide range of commodities, services, occupations etc, and where they have been published. Some 1000 topics are covered and about 2500 sources identified with an index for easy use. It covers all official and significant non-official sources published during the last ten years.

The following publication is compiled by the Department of Employment.

j) *Employment Gazette*. Published monthly, it is a summary of statistics on: employment, unemployment, numbers of vacancies, overtime and short time, wage rates, retail prices, stoppages. Each publication includes one or more ‘in-depth’ article and details of arbitration awards, notices, orders and statutory instruments.

The following publication is compiled by the Department of Industry.

k) *British Business*. Published weekly, the main topics are production, prices and trade. It includes information on: the Census of Production, industrial materials, manufactured goods, distribution, retail and service establishments, external trade, prices, passenger movements, hire purchase, entertainment.

Other important business publications include: HSBC Holdings plc Annual Review, NatWest Bank Quarterly Review, Lloyds TSB Annual Report, Barclays Review (quarterly), International Review (Barclays, quarterly), Three Banks Review (quarterly), Journal of the Institute of Bankers (bi-monthly), Financial Times (daily), The Economist (weekly) and The Banker (monthly).

24. Summary

a) Data that are used for the specific purpose for which they are collected are called primary data. Secondary data is the name given to data that are being used for some purpose other than that for which they were originally collected.

b) A census is a survey which examines every member of the population. Three important official censuses are the Population Census, the Census of Distribution and the Census of Production.

c) A sample is a relatively small subset of a population with advantages over a census that costs, time and resources are much less. The main disadvantage is that of acceptability by the layman.
d) Bias is the tendency of a pattern of errors to influence data in an unrepresentative way. Bias can be due to selection procedures, structure and wording of questions, interviewers or recording.
e) A sampling frame is a structure which lists or identifies the members of a population.
f) Random sampling numbers are tables of randomly generated digits, used to ensure that the selection of the members of a sample is free from bias.
g) Simple random sampling is a technique which ensures that each and every member of a population has an equal chance of being chosen for the sample.
h) Stratified random sampling ensures that every significant group in the population is represented in proportion in the sample using a stratification process. An extensive sampling frame is needed with this method.
i) Systematic (quasi-random) sampling involves selecting a random starting point and then sampling every n-th member of the population. The value of n is chosen based on the size of sample required. It can be biased if certain recurring cycles exist in the population, but can often be used where no sampling frame exists.
j) Multi-stage (quasi-random) sampling is normally used with homogeneous populations spread over a wide area. It involves splitting the area up into regions, selecting a few regions randomly and confining sampling to these regions alone. It is cheaper than random sampling.
k) Cluster (non-random) sampling involves exhaustive sampling from a few well chosen areas. It is a cheap method, useful for populations spread over a wide geographical area where no sampling frame exists.
l) Quota (non-random) sampling normally involves teams of interviewers who obtain information from a set quota of people, the quota being based on some stratification of the population. It is commonly used in market research.
m) The precision of some statistic obtained from a sample can be measured by describing the limits of error with a given degree of confidence.
n) Some factors involved in determining the size of a sample are: money and time available, survey aims, degree of precision or number of sub-samples required. Generally, the larger the sample the better, but small samples can give relatively accurate information about a population.
o) Main methods of primary data collection are:
   i. Individual (personal) interview.
   ii. Postal questionnaire.
   iii. Street (informal) interview.
   iv. Telephone interview.
   v. Direct observation.
p) The main points in questionnaire design are: questionnaire to be as short as possible; questions to be simple, non-ambiguous, non-technical, not to be

personal or offensive and not to involve calculations or tests of memory; answer categories to be given where possible; questions asked in a logical order.

q) Secondary data can be used where the facilities for your own survey are not available or where the secondary data gives all the information you require. Disadvantages are: data might not be of an acceptable quality or out-of-date; geographical or strata coverage may not be appropriate; there may be differences in the meaning of terms.

r) Some of the main sources of external secondary data are contained in the following official publications:

Annual Abstract of Statistics; Monthly Digest of Statistics;
Financial Statistics; Economic Trends;
Regional Trends;
United Kingdom National Accounts (Blue Book);
United Kingdom Balance of Payments (Pink Book);
Social Trends; Employment Gazette;
British Business.

25. Student self review questions

1. Explain the difference between primary and secondary data. [2]
2. Give the meaning of a census and give some examples of official censuses. [3]
3. What are the major factors involved when deciding between a sample and a census? [3,4]
4. Describe what bias is and give some examples of how it can arise. [5]
5. Give at least four examples of a sampling frame. [6]
6. What is a random sample? [7]
7. What is quasi-random sampling and under what conditions might it be used? [7]
8. What are random sampling numbers and how are they used in simple random sampling? [8,10]
9. What does the term ‘stratification of a population’ mean and how is it connected with stratified sampling? [11,12]
10. What are the advantages and disadvantages of stratifed sampling when compared with simple random sampling? [12]
11. What is the difference between homogeneous and heterogeneous populations? [11,13]
12. Give an example of a situation where a systematic sample could be taken:
    a) where a sampling frame exists;
    b) where no sampling frame exists. [13]
13. What are the differences between multi-stage and cluster sampling methods? [14,15]
14. In what type of situation is quota sampling most commonly used and what are its main merits? [16]
15. How can the precision of a sample estimate be expressed? [17]
16. What are the factors involved in determining the size of a sample? [18]
17. List the main methods of collecting primary data [19]

18. What are the advantages and disadvantages of a postal questionnaire over a personal interview? [19]
19. Give some important considerations in the design of a questionnaire. [20]
20. Under what conditions might secondary data be used and what are its possible disadvantages compared with the use of primary data? [21]
21. Name some of the major official statistical publications. [22]

26. Student exercises

1. *MULTI-CHOICE*. Which one of the following is NOT a type of random sampling technique:

   a) Quota sampling
   b) Systematic sampling
   c) Stratified sampling
   d) Multi-stage sampling

2. *MULTI-CHOICE*. A 2% random sample of mail-order customers, each with a numeric serial number, is to be selected. A random number between 00 and 49 is chosen and turns out to be 14. Then, customers with serial numbers 14, 64, 114, 164, 214, ... etc are chosen as the sample. This type of sampling is:

   a) simple random b) stratified c) quota d) systematic.

3. A large company is considering a complete reshaping of its pay structures for production workers. What data might be collected and analysed, other than technical details, to help the management come to a decision? Consider both primary and secondary sources.
4. What factors would govern the use of a sample enquiry rather than a census if information was required about shopping facilities throughout a large city.
5. *MULTI-CHOICE*. A sample of 5% of the employees working for a large national company is required. Which one of the following methods would provide the best simple random sample?

   a) Wait in the car park in a randomly selected branch and select every tenth employee driving in to work .
   b) Use random number tables to select 1 in 20 of the branches and then select all the employees.
   c) Select a branch randomly and use personnel records to choose 1 in 20 randomly.
   d) Select 5% of all employees from personnel records at head office randomly.

6. Suggest an appropriate method of sampling that could be employed to obtain information on:

   a) passengers' views on the adequacy of a local bus service;
   b) the attitudes to authority of the workforce of a large company;
   c) the percentage of defects in finished items from a production line;
   d) the views of Welsh car drivers on the wearing seat belts;
   e) the views of schoolchildren on school meals.

7. A national survey has revealed that 40% of non-manual workers travel to work by public transport while one-half use their own transport. For all workers, 47.5% use public transport and one in every ten use methods other than their own or public transport. A statistical worker in a large factory (which is known to have about

three times as many manual workers as non-manual workers) has been asked to sample 200 employees for their views on factory-provided transport. He decides to take a quota sample at factory gate B at five o'clock one evening.

a) How many manual workers will there be in the sample?
b) How many workers who travel to work by public transport will be interviewed?
c) Calculate the quota to be interviewed in each of the six sub-groups defined.
d) Point out the limitations of the sampling technique involved and suggest a better way of collecting the data.

8. The makers of a brand of cat food 'Purrkins' wish to obtain information on the opinions of their customers and include a short questionnaire on the inside of the label as follows:
    1. Do you like Purrkins?
    2. Why do you buy Purrkins?
    3. Have you tried our dog food?
    4. What amount of Purrkins do you normally buy?
    5. When did you start using Purrkins?
    6. What type of house do you live in?

    Criticise the questions.
9. Design a short questionnaire to be posted to a sample of customers to obtain their views on your company's delivery service.
10. A proposal was received by the Local Authority Planning Office for a motel, public house and restaurant to be built on some private land in the city suburbs. Following an article by the builder in the local paper, the office received 300 letters of which only 28 supported the proposal. What conclusions can the Planning Officer draw from these statistics? Describe what action could be taken to gauge people's views further.

Answers to student exercises

Chapter 2 – Sampling and data collection

1. a) is the correct answer.

   c) is a sophisticated and expensive form of random sampling and b) and d) are quasi random sampling techniques.
2. (d) is correct. Picking a random starting point and then choosing every nth item to coincide with the proportion desired is the precise way systematic samples are structured.
3. Primary: current average pay and estimated new average pay; workers attitudes to new scheme; pay structures in similar companies; official trade union views. Secondary: hours worked and average pay in this industry and over all industries.
4. General: available time and manpower; complexity of enquiry; a census might be feasible if information can be obtained by observation only. Probably a sample would suffice unless the survey was official. Is there a sampling frame available?
5. (d) is correct, since it is the only one that gives each employee of the whole company an equal chance of being chosen.
6. a) Probably cluster

   b) Stratified would be essential, due to the nature of the enquiry (personnel records would be a good sampling frame)

   c) Systematic

   d) Multi-Stage/Cluster

   e) Simple random (from school records sampling frame).
7. a) 150 (b) 95

   c)

	Own	Public	Other	TOTAL
Manual	60	75	15	150
Non-manual	25	20	5	50
TOTAL	85	95	20	200

   d) Better to spread sample over more than one location and occasion, since: factory gate B might be used by only certain types of workers; shift workers who finish before 5 o'clock will not be considered. Also, workers might resent the inconvenience and the statistical worker would find extracting the information difficult with so many people moving quickly. It will also be very difficult to count moving cars and interview people at the same time. A short simple census would be much better (random sampling would not be appropriate here).
8. 1) A pointless question 2) Satisfactory 3) Which dog food?

   4) Exactly what does 'amount' mean and over what period of time?

   5) Difficult memory question (how accurate does the respondent have to be?) Better is one of, say, four boxes to be ticked.

   6) Better to give a set of house categories and ask respondents to choose one.

10. People who are against the scheme are much more likely to write in. Thus no conclusions can be drawn. An unbiased sampling technique (based in the area immediately surrounding the proposed scheme) would be appropriate.

Chapter 3 – Data and their accuracy

1. c) is the correct answer. Counting the number of times an event occurs will always give a discrete (precise) value. Times, weights and ages can never be calculated precisely, only approximated. a), b) and d) are examples of continuous data.
2. a) is the correct answer. The largest value that 8.2 can take is 8.25 and the largest value that 16 can take is 16.5, giving 24.75 as the largest value that their sum can take.
3. a) Univariate, variable='time to complete job', continuous, numeric.
   b) Bivariate, variable 1='job title', non-numeric, discrete;
   variable 2='age', numeric, continuous.
   c) Bivariate, variable 1='location', non-numeric, discrete;
   variable 2='no. of employees', numeric, discrete.
   d) Univariate, variable='dept. name', non-numeric, discrete.
   e) Multivariate (5-variable):
   variable 1='average wage', numeric, discrete;
   variable 2='manual/non-manual', non-numeric, discrete;
   variable 3='sex', non-numeric, discrete;
   variable 4='industry', non-numeric, discrete;
   variable 5='year', numeric, discrete.
4. (a) £148 360 (b) 23 000 tons (c) 3.2 mm (d) £16 (e) 30 months (f) £18 600
5. b) is the correct answer. The form of sampling used has no particular bearing on the way data is measured and thus its accuracy. Therefore c) and d) are misleading statements.
6. (b) is correct. Kappa range is (95000, 105000) and Lambda (180000, 220000). Thus, range of joint stock is (275000, 325000) which has a possible highest error of 25,000 of the estimated 300,000 valuation = 25000/300000 = 0.083 = 8.3%
7. (a) [192,206] (b) [56,96] (c) [159.71,172.15] (2D)
   (d) [457.54,502.38] (2D) (e) [0,0.75] (2D)
8. (a) £600
   (b) [0,£1320]. Buy = [180,220]. Per item worst = Buy at £16, Sell at £16, Profit=0; Per item best = Buy at £14, Sell at £20, Profit=£6. Thus profit range = £[180×0, 220×6].
9. (a) [1776,2028] (b) (i) [£3463.20,£4157.40] (ii) [£5239.20,£6185.40]
   (c) [£1081.80,£2722.20] (d) £3800; min profit = 28.5%, max profit = 71.6%
10. [104 269,105 816] in £000.
11. (a) [53,107] (b) 80

Chapter 4 – Frequency distributions and charts

1.

Number of vans unavailable	0	1	2	3	4	5	6	7	8	Total
Number of days	12	21	11	9	2	3	1	0	1	60

Comments: The most common number of vans unavailable was 1 and on only 12 occasions were all vans available. On most days there were not more than 3 vans unavailable and at no time was there more than 8 vans unavailable.

2. (a) Since the data is discrete (counted), the limits of separate classes should not meet.
   (b) There are too few classes given.
   (c) The distribution of frequencies is not good.
   (d) The last two classes are out of order.
   (e) A better structure would be to have classes such as: 500–509, 510–519, 520–524, 525–529, 530–534, 535–539, 540–559, 560–579.

3. (a)

	Limits		Boundaries			
Class	lower	upper	lower	upper	Width	Mid-point
0 to 4	0	4	0	4.5	5	2
5 to 9	5	9	4.5	9.5	5	7
10 to 19	10	19	9.5	19.5	10	14.5
20 to 29	20	29	19.5	29.5	10	24.5
30 to 49	30	49	29.5	49.5	20	39.5

   (b) 12(2)+28(7)+9(14.5)+7(24.5)+2(39.5) = 601.

4.

Diameter of bolt	Number of bolts
1.90 – 1.94	3
1.95 – 1.99	9
2.00 – 2.04	17
2.05 – 2.09	10
2.10 – 2.14	9
2.15 – 2.19	5
2.20 – 2.24	5
2.50 and over	2

6. (b) 4143 (c) Total number of car-hours per day = 400 ×12 = 4800.
   Thus utilization = $\dfrac{3485}{236}$ = 86%

8. (a) 19 (b) 14 (c) 238 hours (approx)

9.

0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
1	1	8	8	12	25	19	14	9	3

10.

Repayments (under, £000)	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
% of home owners	2	17	42	69	87	95	98	100

   (i) 30 (ii) 38

11. (a)

Cumulative % repayments	0.3	6.9	25.3	53.1	76.9	89.9	95.6	100
Cumulative % number of home owners	2	17	42	69	87	95	98	100

   (c) 37

12.

Cumulative % net output	11	37	49	65	78	100
Cumulative % number of firms	40	72	87	94	98	100

   The largest 20% of firms account for approximately 57% of all net output.

13.

Cumulative % income before tax	1	6	15	35	60	72	82	89	100
Cumulative % number of incomes	4	16	30	56	80	89	95	98	100

Chapter 5 – General charts and graphs

3. Line diagram or simple bar chart. The latter should have each bar 'broken' in some way to emphasise the break of scale, making the former preferable. The shifts appear to be in cycles of three.
4. Multiple line diagram and multiple bar chart. The former is preferable, particularly since the three variable values are conveniently separate.
5. Neither; they both show different aspects of the data. Both have their place, depending on whether absolute values and totals or relative comparisons are more important.
7. Radius 2 = 1.17 × radius 1; radius 3 = 1.25 × radius 1. Angles (in degrees):
   Circle 1 = 50, 81, 166, 63;
   Circle 2 = 49, 62, 177, 71; Circle 3 = 47, 53, 171, 90.

Chapter 6 – Arithmetic mean

1. (a) 80.4 (1D) (b) 0.507 (3D)   2. (a) 19.91 (2D) (b) 4.1 (1D)
3. (a) is correct. 'Expected' = mean = [0(6)+1(3)+4(2)+4(3)+2(4)+1(5)]/20 = 1.8.
4. 25.2 (1D)   5. 37.04 yrs (2D)
6. 425.0012 gms (4D). The consumer is getting reasonable value since the mean is (just) over the advertised contents weight of 425 gms.
7. (a) 23.625 (b) 40   8. (a) 16.7 (b)(i.) 24.3 (ii.) 20.5 (c) 62
9. d) is incorrect. The mean CAN always be calculated, no matter how large the set of values is.

Chapter 7 – Median

1. (b) is correct. The items in size order are: 35 35 36 36 36 37 38 40 40 42 43 and the middle item is the 6^{th}, which is 37.
2. (a) 3.75 (b) 78.5
3. (a) 22 (b) Extreme values are present at the lower end of the distribution. (c) The bags would probably be packed to some nominal weight, which would mean that the actual weights would be fairly symmetrically distributed. Thus the mean would be an ideal average.
4. 126.75. A class width of 9 is not a very desirable one from the point of view of an observer attempting to comprehend the general nature of the data. A much better class structure would be 90–99, 100–109, etc.
5. 1.79 hours (2D).   6. 580 hours.

Chapter 8 – Mode and other measures

1. (a) 11 (b) 2 and 3 (bi-modal) (c) 15
2. 21.8 (1D). Since the distribution is fairly well skewed, the mode is giving a value that is too high for practical purposes. The median is probably more suitable here.
3. Mode=1. This type of data is ideally suited to a modal average, since in normal circumstances the most typical number of children can be put to more practical use than a non-typical mathematical value given, for instance, by the mean.
4. (c) is incorrect. Refer to Figure 3 for the relative positions of the mean, median and mode in a right-skewed distribution.
5. (a) 2972 (b) 2769 (c) 2744 (d) 456 (e) 7. Distribution is moderately skewed.
6. 3;2.88;2.77.
7. 75.4 customers per hour.
8. 3.2%
9. 8.21%
10. (a) 58.3 (b) 56.3
11. (a) £20/sq yd (b) £32/sq yd.

Chapter 9 – Measures of dispersion and skewness

1. 25; 8.375
2. Before: mean=£115.50; md=£22.90. After: mean=£98.50; md=£30.20.
3. (a) 4.1 (1D) (b) 0.81 (2D) (c) 4 (d) 0.77 (2D)
4. 5.0 (1D)

Chapter 10 – Standard deviation

1. 1.63 (2D)
2. (c) is correct. All values have been converted to percentages by multiplying by 4. The standard deviation is measured in the same units as the petty cash amounts, thus the new mean and standard deviation also should be multiplied by 4.
3. (a) 6.6; 6; 8; 1.9 (1D) (b) 0.3 (1D) (c) Range/6 = 1.3 (1D). Sd is 1.9 (larger than 1.3) since the distribution is right skewed.
4. (c) is correct. CV = (mean/sd)×100% = 25%.
5. (a) 36.32 (2D); 9.95 (2D). (b) 16.42 and 56.22.
6. (a) 0.16712; 0.00221 (b) cv(1)=1.32%; cv(2)=1.82% (c) 0.16571 (d) New mean increased to 0.16800. Sd unaltered at 0.00300. New cv(2)=1.79%. Second sample still more variable.

Chapter 11 – Quantiles and the quartile deviation

1. (a) is correct. All the others are measured in the same units as the data.
2. Median=30; Q1=28; Q3=32; qd=2
3. 1; 1
4. (a) 35.07; 9.74 (b) 0.07
5. (b) is correct. The diagram shows a 'less than' ogive and C corresponds to a point 600/800 = 75% along the distribution of values.
6. (a) 10.90; 11.98 (b) Only 30% of bulbs last longer than 11.98 days.
7. Classes are 0.5–1.5, 1.5–2.5, ... etc. Median=4.15 wks.

Chapter 12 – Linear functions and graphs

1. (b) is correct.
2. (a) 4;13 (b) 3;–12 (c) 2;1.5 (d) 0.25;0.5
3. (Graphs NOT given in these answers)
4. (a) 1 (b) –6 (c) 0.1
5. (a) $y = 1 + x$ (b) $y = 6 - x$ (c) $y = 0.5x - 1$
6. $y = 2x - 4$
7. (a) is correct. The relationship can be re-arranged into the form $y = 4 - 2x$ which shows the gradient is –2.
8. (c) is correct. Only equation (c) satisfies the two points x=12, y=2 and x=8, y=0

Chapter 13 – Regression techniques

1. Mean value = (7,5); Estimate = 6.7 (approx).
2. $y = 0.51 + 0.64x$; Estimate = 6.9 (approx).
3. $y = -0.6 + 0.9x$; 48.9
4. $y = 1.94x + 10.83$
5. (d) is correct. Taking the regression line in the form $y = mx + c$, c = 6, since the lines meets the y-axis at 6. Also the orientation of the line shows that the gradient (m) is negative with ratio –6/6 = –1.
6. x=capacity, y=price. $y = 1236.9 + 3.0192x$.
7. (a) There is a clear case of independent versus dependent variable here, since overtime worked will presumably depend on how much work there is to do (i.e. how many orders have been received). Thus x=orders and y=overtime. Line is $y = -2.26 + 0.40x$ and $y(100)$=38 (i.e. 38 overtime hours are needed to support 100 orders).
   (b) We require the number of orders that corresponds to 35 hours total overtime, which is obtained by substituting y=35 into the above line. The value of x obtained (number of orders) is the least that would be acceptable as a criterion for taking on a new employee.

Chapter 14 – Correlation techniques

1. 0.98(2D)
2. (a) is correct. Value +1 is perfect positive correlation and –1 is perfect negative correlation. No correlation has value zero.
3. (a) is correct. Remember that positive correlation shows a pattern stretching from bottom left to top right. Also, the stronger the correlation is, the more the points resemble a straight line.
4. 0.60
5. 0.97. This shows that 97% of the variation in profit (before taxation) is due to variations in turnover. Clearly a result to be expected, since no other significant factors should be expected.
6. r=0.93. A high positive correlation coefficient seems appropriate. A cost on output regression line could then be extrapolated with some degree of confidence.
7. (a) 0.49 (b) 0.33
8. r'=0.51. Only a moderate degree of positive correlation, and with the coefficient of determination (r'^2) = 0.26, this means that only 26% of the movement of the share price can be attributed to the FT Index. Thus the FT Index can be considered as only an approximate indicator for the particular share price.

9. r'=0.67. A moderate level of agreement. They both clearly agree on the best and the worst suppliers and in between there is no strong measure of disagreement. No particular conclusions about quality of suppliers can be deduced since the two managers would be assumed to weigh various attributes of suppliers differently, according to their own needs and functions.
10. (a) is incorrect. Data must always be numeric for the product moment correlation coefficient to be calculated.

Chapter 15 – Time series model

1. Extreme weather (storms, heat waves); stock shortages; special events (fairs, shows); new competition in the area; bank holidays; national strikes; special sales offers; 'flu epidemics.
2. Company sales are highly seasonal, with the peak during the summer. Autumn sales are beginning to catch the value of sales in the winter quarter. The general trend in sales is upwards.

Chapter 16 – Time series trend

1. 15.4, 15.2, 15.0, 14.8, 14.6, 14.4, 14.2, 14.0, 13.8, 13.6
2. (a) is correct. A 4-point centered average will always leave two time points at either end of the data without a plotted point.
3. (a) 9.7,14.0,11.7,11.3,8.3,10.3,15.0,12.7,12.7,9.3,11.3,16.7,14.3 (1D)

   (b) 10.8,11.0,11.2,11.2,11.6,11.8,12.0,12.2,12.4,13.0,13.2

   The moving average of period 5 is the correct one to use for the trend, since the data cycle over groups of 5 exactly.
5. The moving average trend, beginning at Year 4 is (to nearest unit):

   323, 328, 332, 339, 346, 357, 366, 375, 384, 390, 392.
6. 3.3, 3.4, 3.5, 3.6, 3.7, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 4.5

Chapter 17 – Seasonal variation and forecasting

1. (a) Seasonal factors: 13.8, 23.8, –41.3, 3.8

   Seasonally adjusted figures: 126, 126, 136, 131, 136, 146, 146, 161, 166, 166, 176, 171

   (b) Seasonal factors: 1.09, 1.17, 0.71, 1.02

   Seasonally adjusted figures: 127, 128, 137, 126, 137, 147, 145, 168, 166, 165, 171, 182
2. (a) Season 1 = –2.2; season 2 = 0.2; season 3 = –1.5; season 4 = 10.6; season 5 = –7.1

   (b) 10.2,10.8,11.5,10.4,11.1,11.2,11.8,11.5,12.4,12.1,12.2,12.8,12.5,15.4,13.1

   (c) 11.7, 14.4, 12.9, 25.2, 7.8
3. (a) 0.78, 0.95, 0.98, 1.29 (b) 318, 302, 319 (to nearest unit)

   (c) 289, 301, 403 (to nearest unit)

4 (a) –0.8, 0.5, –0.6, 0.9 (b) 3.8, 5.2, 4.2, 5.8

5. (c) is correct. The model is: y = t × S or Sales = Trend × Seasonal.

   ie 1600 = Trend × 0.8

   Thus, trend value = 1600/0.8 = 2,000.

6. (a) is correct. The seasonal (proportion) values must add to 4, which gives Q4 seasonal value as 0.5. Alternatively, the percentage seasonal variation should add to zero, giving Q4 seasonal variation as –50%.
7. (a) is correct. Trend for Q1 is known as 2,000 (from Q5). Similarly, Q2 trend = 4400/2 = 2,200 and Q3 trend = 1680/0.7 = 2,400. Clearly, Q4 trend = 2,600. Thus Q4 sales = trend × seasonal factor = 2600 × 0.5 = 1,300.
8. Seasonally adjusted values:
   Additive: 6.4, 9.1, 8.6, 8.4, 8.9, 8.1, 8.2, 8.8, 8.2, 9.2, 8.9, 9.8 (%)
   Multiplicative: 7.1, 9.2, 8.6, 8.2, 8.8, 8.1, 8.2, 8.8, 8.3, 9.3, 9.0, 10.2 (%)
   Forecast for 1983, quarter 1 = 13% (additive and multiplicative)
   The multiplicative model would be better since figures are percentages.

Chapter 18 – Index relatives

1. (c) is correct, since $120*(1.05)^3$ = £162.07.
2. Price relative = 90; quantity relative = 130; expenditure relative = 117.
3.

	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct
(a)	100	88.7	90.1	73.2	76.1	102.8	111.3	96.5
(b)	110.9	98.4	100	81.3	84.4	114.1	123.4	107.0
(c)	97.3	86.3	87.7	71.2	74.0	100	108.2	93.8

4.

	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov
Fixed base	100	103.8	108.2	109.4	108.4	106.1	103.5	104.2	101.5
Chain base	103.8	104.2	101.1	99.2	97.8	97.5	100.8	97.4	–

5. (a)

	19X1	19X2	19X3	19X4	19X5	19X6	19X7	19X8	19X9
Chain base index	–	105.2	98.0	112.0	114.3	94.5	119.8	102.8	102.0
Amount harvested	–	–	–	587	–	634	760	781	797

6.

	19X0	19X1	19X2	19X3	19X4	19X5	19X6	19X7
Index for firm (19X2=100)	101	96	100	107	98	98	103	107
National index (19X2=100)	90	89	100	104	97	96	100	104

7.

Whole economy	100	102	103	103	104	106	105	107	106	107
Coal and coke	100	157	176	178	189	191	193	196	196	204

8.

Time point	1	2	3	4	5	6	7
Real index	100	99.7	102.1	110.7	111.6	111.9	112.8

9.

	19X5	19X6	19X7	19X8	19X9	19Y0	19Y1
Fixed base	100	99.5	89.2	98.2	99.0	98.3	105.5
Chain base	–	99.5	89.6	110.1	100.8	99.3	107.3

10. (a) is correct. Over the period, inflation has increased by 210/180=16.7% and money wages by 115/100=15%. Thus (i) is correct. Also, 5% compounded for 3 years yields $1.05^3 = 1.157$ or 15.7%. Thus (ii) is incorrect.
11. (b) is correct. By calculating the RVI (see section 13) we obtain $\frac{115}{100}\times\frac{180}{210}\times 100$ which represents a 1.43% decrease.

Chapter 19 – Composite index numbers

1. (a) is correct. 7(130)+3X=10(127) is rearranged to give 3X=360. Thus X=120.
2. (a) 105.4 (b) 104.9
3. I_{Feb}=110.2; I_{Mar}=101.2; Volumes increased overall by 10% from Jan to Feb, then fell by almost the same amount from Feb to Mar.
4. 101.7
5. L_2=110.6; L_3=176.6.
6. (a)

	19X0	19X1	19X2
Wheat	0.99	0.99	0.98
Barley	0.19	0.19	0.24
Oats	2.70	3.15	3.48

(b) L_{19X1}=107.0; L_{19X2}=112.4

(c) P_{19X1}=107.3; P_{19X2}=112.3

7. P_{cost}=107.7; $P_{quantity}$=109.3

Chapter 21 – Interest and depreciation

1. 44; 345	2. 47.4	3. –560	4. 2187; 3280
5. 18.60	6. 70	7. £1650	8. (a) £924.20 (b) £4578.47
9. £873.22	10. 14	11. 24.57%	12. £9929.97

13. (d) is correct. APR is calculated at 1% compounded for 12 periods (of 1 month). This is $(1.01)^{12}$ = 1.1268 yielding 12.7%.
14. (a) 18.81% (b) 19.25% (c) 19.56%
15. (b) is correct. Using the depreciation formula in section 22 gives $5 = 46(1–i)^3$. This can be rearranged to give i = 0.4772, giving the average rate as 47.7%.
16. £35895
17. (a) is correct. £20,000 × $(0.78)^3$ = £9,491 and £20,000 × $(0.82)^3$ = £11,027.
18. (a) £55188.71 (b) 17.57%
19. £4774.42

Chapter 22 – Present value and investment appraisal

1. (a) £986.27 (b) £1134.85 (c) £3583.79
2. PV of £10000 in 2 years is £9053.99. Therefore pay £9000 now.
3. £15686.62
4. PV of debt = £2524 + o/heads of £250 gives total cost of £2774. So the minimum selling price = £277.40 per set.
5. The real comparison is between £800 now or £1000 in 1 year. This represents a discount rate of 25%, which is higher than any standard investment rate. Hence, £1800 cash is well spent here!
6. NPV = £5366.80.
7. At rate 18%, NPV is £14069. Purchase is recommended.
8. At 18%, NPV = £14069; at 25%, NPV = –£3359. IRR estimate = 23.7%.
9. (a) NPV1 = £11314.70; NPV2 = £12816.00. Choose project 2 on NPV.
   (b) NPV1 = –£232.20; NPV2 = –£1202.50. IRR1 = 19.9%; IRR2 = 19.6%. Choose project 1 on IRR.
   (c) The two projects are different in scale and also have different flow patterns. (Note however that with a discount rate of 20%, the NPV criterion would choose project 1 since it has the least deficit.)
10. (a) –£3490 (b) The value of the project will not be enough to repay the loan.

Chapter 23 – Annuities

1. (a) £2933.30 (b) £3167.96
2. £317.62
3. (d) is correct. Calculation = £17,000/(0.06) = £283,333.
4. (a) £9124.69 (b) £14419.43 (c) £21739.13
5. (b) is correct. Use the formula in section 11 with P=£200,000, n=15 and i=0.06
6. (a) £13224.31

   (b)

Year	1	2	3	4	5
Interest paid (£)	7062.00	6045.22	4860.67	3480.67	1872.97

7. (a) £2196.69 (b) £183.06 per month can be invested as an ordinary annuity at 0.75%/mth to yield £2289.64 at year end. Therefore excess = £92.95 = 0.5% of original principal.
8. Payment is £3686.45
9. Depreciation charge is £2490.56
10. £7767.23; £14784.46

Chapter 24 – Functions and graphs

1. (a) is correct. $\frac{20}{0.2}\times\frac{1}{r}=\frac{20}{0.2}\times\frac{1}{100}=\frac{20}{20}=1$
2. x=0.82; y=9.53
3. (a) x=100, y=200 (b) x=3, y=4 (c) x=–2, y=4 (d) x=12, y=–12
4. Plotted points:

x	–4	–3	–2	–1	0	1	2
y	9	0	–5	–6	–3	4	15

   (i) $-3 < x < 0.5$ (ii) $x < -3$ and $x > 0.5$

5. Plotted points:

x	–1	0	1	2	3	4	5
$14-3x$	17	14	11	8	5	2	–1
$2x^2-12x+14$	28	14	4	–2	–4	–2	4

   x=0, x=4.5

6. Plotted points:

x	–2	–1	0	1	2	3	4
$2x^2-4x-5$	11	1	–5	–7	–5	1	11
$9+5x-x^2$	–5	3	9	13	15	15	13

   x=–1, x=4

7. (a) x^3-4x^2+x+6

   (b) Plotted points:

x	–1	0	1	2	3	4
x^3-4x^2+x+6	0	6	4	0	0	10

   Curve meets x-axis at x=–1, x=2 and x=3.

   (c) x=0 and x=1.5

8. Plotted points:

x	1	1.25	1.5	1.75	2	2.25	2.5
y	–0.5	0.19	0.5	0.44	0	–0.81	–2

   (i) 1200 and 2000 (2S) (ii) max profit = £520, production level = 1600 (2S)

9. (a) Plotted points:

x	1	2	3	4	5	6
y	315	220	191	180	175	173

   (b) This gives total running costs per week for x lorries (c) £45500

Chapter 25 – Linear equations

1. (a) is correct. To solve, equate to give $3x–2 = x+2$ yielding $x=2$, $y=4$.
2. (a) $x=6$ (b) $x=13$ (c) $x=-1$ (d) $x=5$ (e) $x=5$ (f) $x=0.5$ (g) $x=-2$ (h) $x=-2.5$ (i) $x=-15$ (j) $x=2$ (k) $x=5$
3. $x=3$ 4. $y=1.5$
5. (a) $x=3, y=1$ (b) $x=3, y=5$ (c) $x=1, y=2$ (d) $x=4, y=1$ (e) $x=7, y=3$
6. (a) $x=5, y=3$ (b) $x=3, y=1.5$ (c) $x=y=4$ (d) $x=-0.1, y=0.4$
7. 6 8. (a) $10(0.9)x+3y=37.5$; $7x+4y=40$ (b) $x=2, y=6.5$
9. 45A and 15B tables 10. (a) $x=2, y=-1, z=5$ (b) $x=2, y=3, z=-1$ (c) $x=-1, y=10, z=5$
11. 25X, 5Y and 20Z products.

Chapter 26 – Quadratic and cubic equations

1. (a) –3,4 (b) –1/3,5/2 (c) –2,2 (d) 0,0.4 (e) –1.5,8 (f) 3,4 (g) 2,3.5 (h) –2,2.5
2. (d) is correct. For the formula: a=1, b=–2 and c=–24. Inside the root, the calculation needs to be made carefully as: $\sqrt{[(-2)^2 - 4(1)(-24)]} = \sqrt{[4 + 96]} = \sqrt{100} = 10$.
3. (a) 1.5,2.5 (b) 2.38,4.62 (c) 3,4 (d) No solutions (e) –0.72,1.39 (f) 0.37,3.63 (g) 3.05,7.06 (h) –1.32, 0.57
4. (a) Plotted points:

x	0	1	2	3	4	5	6	7	8
y	4.5	1	–0.5	0	2.5	7	13.5	22	32.5

   (b) (i) 1.5,3 (ii) 0.4,4.1 (iii) 1.3,4.2 (iv) 0.4,7.2
5. (a) –4.65,0.65 (b) –2.58,0.58 6. 0,1,2
7. Plotted points:

x	–6	–5	–4	–3	–2	–1	0	1	2	3
$y=2x^3+8x^2-18x-25$	–61	15	47	47	27	–1	–25	–33	–13	47
$y=2x+20$		10				18			24	

8. (a) Plotted points:

x	2	2.5	3	3.5	4	4.5	5
$x^3-10.55x^2+36.4x-40.8$	–2.20	–0.11	0.45	0.24	0	0.49	2.45

   solutions = 2.55,4,4
   (b) solutions are –0.5,0.5,3
   (c) Plotted points:

x	–2	–1	0	1	2	3	4	5
$-20+8x+12x^2-3x^3$	36	–13	–20	–3	20	31	12	–55

   solutions = –1.4,1.1,4.3
9. –0.97

Chapter 27 – Differentiation and integration

1. (a) $8x$ (b) $12x^3+4x$ (c) $60x–10$ (d) $8 - \frac{2}{x^2}$ (e) $2x+3$ (f) $4.5x^2$ (g) $2 - \frac{4}{x^2}$
2. (a) $\frac{dy}{dx} = 10x-2$, $\frac{d^2y}{dx^2} = 10$ (b) $\frac{dy}{dx} = -10+12x-6x^2$, $\frac{d^2y}{dx^2} = 12-12x$

   (c) $\frac{dy}{dx} = 9x^2+4$, $\frac{d^2y}{dx^2} = 18x$
3. Turning point is a minimum at $x=2, y=-8$
4. $x=0.33$, $y=3.81$ (max) and $x=5$, $y=-47$ (min)

5. (a) $3x^3+C$ (b) $0.5x^4+C$ (c) $2x^4-x^3+4x^2-10x+C$ (d) $\frac{-1}{x^2}$ 6. $y=2x^2-3x+6$

7. (a) $P=11x-x^2-24$ (b) 8000 (i.e. $x=8$) (c) 5500 (i.e. $x=5.5$) (d) £625 (i.e. $P=6.25$)

Chapter 28 – Cost, revenue and profit functions

1. (b) is correct. C = 4 + 0.8(2) = 4 + 1.6 = 5.6 (000) = 5,600.
2. (d) is correct.
3. (a) $P=18x-20-4x^2$ (b) 225 units produced give a total profit of £250 (c) A loss of £6000
4 Cost-minimising yearly level of production is 4309 items. Total cost is £16661.27
5 p_r(£)=13.75–0.175q, where q is the quantity demanded 6. £8100
7. (a) £2.50 (b) (i) $\frac{2000}{x}+2$ (ii) $0.5-\frac{2000}{x}$ (c) 4000 units (d) 5000 units (e) 2000 units
8. (a) $8.9-0.0005x$ (b) $7.1x-0.0005x^2-15000$ (c) £10,205 at a sales level of 7100 (d) £5.35

Chapter 29 – Set theory and enumeration

1. (a) False. (b) False, C is a subset of A. (c) True. (d) False, smallest value must be n[A]=7. (e) False. (f) True (g) True.
2. (a) {a,b,c,d,e,f} (b) {a,b,c,d,e,g,h} (c) {c,d,e} (d) {c,d,e} (e) {a,c,d,e,f,g,h} (f) {a,c,d,e} (g) {a,b,f,g,h,i} (h) {b,i} (i) {a,b,f,g,h,i}
3. (a) U = {r,s,t,u,v,w,x,y} = set of all products stocked.
   (b) {r,t,v,w,x} = set of products bought by either C or D. (c) E∩B = E = {r,v,w,x} = set of products bought by both B and E. Also, all products bought by E are also bought by B (E is a subset of B).
   (d) {s,u,v,w,y} = products that are never bought by C.
   (e) A∪C = {r,s,t,v,x} and B′ = {s,u,y}. Thus (A∪C)∩B′ = {s} = the products bought by either A or C that are never bought by B.
   (f) {r} = the products bought by all regular customers.
   (g) {u,y} = products never bought by any of the regular customers.
4. ab=12, a=16. 5. 5 6. (a) 5 (b) 158 (c) 72 (d) 42 (e) 10
7. (a) 19 (b) 15 (c) 3 (d) 2

Chapter 30 – Introduction to probability

1. b;c 2. c generally; e 3. (a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{5}{6}$
4. (b) is correct. There are 36 combinations altogether and of these just 6 (6 and 1, 1 and 6, 5 and 2, 2 and 5, 3 and 4, 4 and 3) add to 7. Thus probability $=\frac{6}{36}=\frac{1}{6}$.
5. (d) is correct. All three systems must fail with probability $\left(\frac{1}{100}\right)^3$.
6. (a) 0.04 (b) 0.54 (c) 0.42
7. (a) 0.12 (b) 0.03 (c) 0.03 (d) 0.913 (3D) (e) 0.319 (3D)
8. (a) 0.72 (b) 0.92 (c) 0.68 9. (a) 0.023 (b) 0.045 (3D)
10. (a) 0.9 (b) 0.921 (3D) (c) 689 (to nearest unit)

Chapter 31 – Conditional probability and expectation

1. (a) £32,000 (b) 0.16, 0.52, 0.28, 0.04 2. 10.85
3. 36.25 days (using EXPECTATION); 30.5 days (using other techniques). The latter is strictly correct.
4. (a) expected profit = £24.50 (b) expected profit = £18.50. 200 cauliflowers should be bought, since this leads to the largest profit (£27) in a day.
5. (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ 6. (a) 0.42 (b) 0.30 (c) 0.50 (d) 0.71 (2D)
7. (a) 0.553 (b) 0.605 (c) 0.395
8. 0.25 9. (a) 0.80 (b) 0.35 (c) 0.85 (d) 0.60 (e) 0.57 (2D) (f) 0.25 10. $\frac{2}{3}$.

Chapter 32 – Permutations and combinations

1. AB, AC, AD, AE, BC, BD, BE, CD, CE, DE. (10 altogether.)
2. (a) 120 (b) 12 (c) 120 (d) 210 3. (a) 5 (b) 20 (c) 24 (d) 30 4. (a) 6 (b) 4 (c) $\frac{2}{3}$
5. (a) 126 (b) 60 (c) $\frac{60}{126}$ = 0.476 (3D) (d) $\frac{81}{126}$ = 0.643 (3D) (e) $\frac{45}{126}$ =0.357 (3D)

Chapter 33 – Binomial and poisson distributions

1. (a) 30 (b) Trial = selecting a workman; trial success = finding a workman idle; number of trials = 6 (the number of workmen involved).
2.

x	0	1	2	3	4
p	0.316	0.422	0.211	0.047	0.004

3. (a) 2.94; 1.71 (2D) (b) (i) 0.022 (ii) 0.112 (iii) 0.243 (iv) 0.623 (3D)
4.

x	0	1	2	3	4
p	0.1003	0.2306	0.2652	0.2033	0.2006

5. (a) 0.670 (b) 0.062 (c) 0.047 (3D) 6. (a) 0.190 (b) 0.191 (3D)
7. (a) 0.088 (3D) (b) 0.263 (3D) (c) 0.68 8. (a) 0.737 (b) 0.263 (3D) (c) 23.

Chapter 34 – Normal distribution

1. (a) (i) 0.6480 (ii) 0.8888 (iii) 0.9968 (b) (i) 0.1379 (ii) 0.0019 (iii) 0.4483
   (c) (i) 0.1357 (ii) 0.0007 (iii) 0.0268 (d) (i) 0.9962 (ii) 0.7764
2. (a) (i) 0.1314 (ii) 0.0307 (iii) 0.8379 (b) (i) 0.8106 (ii) 0.9846 (iii) 0.1740
   (c) (i) 0.0099 (ii) 0.6591 (iii) 0.3310
3. (a) 0.9772 (b) 0.1587 (c) 0.8185 4. (a) 0.0150 (b) 0.0228 (c) 0.9622
5. (a) (i) 0.0062 (ii) 0.0062 (b) 6
6. (a) 0.0228 (b) 5.56pm 7. 0.1788
8. 0.03
9. Lower limit = 0.9767 cms; upper limit = 1.0233 cms.
10. a) 15.3 ± (1.96)(0.682) = (13.96, 16.64) b) 15.3 ± (2.58)(0.682) = (13.54, 17.06)
11. (0.016, 0.304)
12. a) 20.85; 1.816 b) 20.85±1.34 = (19.51, 22.19) c) 0.714± 0.335 = (0.379, 1.0)

13. $z = -1.35$; no evidence of difference. Therefore yes, sample could have been drawn from given population.
14. $z = -2.81$; evidence of difference. Pay appears to have changed.
15. a) $z = 3.125$; evidence of difference.
    b) $\bar{x} = 832$ gives $z = 2.0$; $\bar{x} = 831$ gives $z = 1.938$. Thus 831 is the largest whole-number value.

Chapter 35 – Linear inequalities

1. (a) To the left, top of line $4x–2y=100$ (b) To the right, top of line $x+3y=20$
   (c) To the left, bottom of line $4x+3y=120$ and to the top of line $y–20$
   (d) To the left, bottom of line $10x+10y=300$ and to the right, top of line $2x+6y=60$ and to the right of line $x=10$.
2. The first inequality should read $x+y<=80$. The four vertices are (40,20), (40,40), (0,80) and (30,20).
3. $x=y=300$; maximum value = 1800. 4. $x=15$, $y=30$; maximum value = 90
5. (a) $x<=400$, $y<=700$, $x+y<=800$, $2x+y<=1000$. Contribution function is $0.4x+0.5y$
   (b) $x=100$, $y=700$. Maximum contribution is £390/day.
6. (a) Process A: $10x+10y<=12000$. Process B: $8x+3y>=4800$. Y: $y>=600$ X: $x<=500$.
   (b) Coordinates for maximum (240,960). Maximum = £158,400. Coordinates for minimum (375,600). Minimum = £121,500. (c) Coordinates for maximum = (500,700). Maximum = £140,000. Coordinates for minimum (375,600). Minimum = £112,500.

Chapter 36 – Matrices

1. (a) 1 (b) 3 (c) 2 (d) –2
2. (a) $\begin{bmatrix} 10 & 3 \\ 5 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 8 & 5 \\ -11 & 5 & 5 \end{bmatrix}$
3. A*B = $\begin{bmatrix} 8 \\ 21 \end{bmatrix}$. A*C is NOT defined. B*C = $\begin{bmatrix} 6 & 2 & 4 \\ 12 & 4 & 8 \\ 3 & 1 & 2 \end{bmatrix}$

   B*A is NOT defined. C*A is NOT defined. C*B = 12 (a 1×1 matrix is a number).

   $$A*B*C = \begin{bmatrix} 24 & 8 & 16 \\ 63 & 21 & 42 \end{bmatrix}$$

4. A*B = total cost per hour for each firm if all machines were utilised. B*C has no practical meaning. C*B = total cost involved for the specified job. A*B*C has no practical meaning.
5. (a) $X = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{matrix} J & K & L \end{matrix} \\ \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 2 \\ 3 & 2 & 2 \end{bmatrix} \end{array}$ (b) $Y = \begin{array}{c} \\ J \\ K \\ L \end{array} \begin{array}{c} \begin{matrix} S & M & F \end{matrix} \\ \begin{bmatrix} 10 & 60 & 10 \\ 10 & 30 & 0 \\ 0 & 10 & 20 \end{bmatrix} \end{array}$

(c) $X^*Y = \begin{array}{c|ccc} & S & M & F \\ \hline A & 20 & 80 & 40 \\ B & 10 & 80 & 50 \\ C & 50 & 260 & 70 \end{array}$ (d) Job C takes 380 minutes.

(e) Here, $Z = \begin{bmatrix} 0.10 \\ 0.40 \\ 0.05 \end{bmatrix}$ and X^*Y^*Z (compatible for multiplication)

$= \begin{bmatrix} 36.0 \\ 35.5 \\ 112.5 \end{bmatrix}$ giving the costs of jobs A, B and C respectively.

6. (a) 3 (b) –11 (c) –10 (d) –33

7. (a) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{bmatrix}$ (c) $\begin{bmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{5}{10} & -\frac{5}{10} \end{bmatrix}$

(d) $\begin{bmatrix} 0 & \frac{1}{11} \\ \frac{1}{3} & -\frac{10}{33} \end{bmatrix}$

8. (a) x=1, y=2 (b) x=2, y=6 (c) p=–2, q=3 (d) x=0.4, y=2.1

9. (a) 3(M*N), in £, is $\begin{array}{c|cc} & S & M \\ \hline X & 3.41 & 2.24 \\ Y & 5.84 & 3.48 \end{array}$

and gives the storage and maintenance cost at warehouses X and Y for each item held over a period of three days (assuming there are no stock movements).

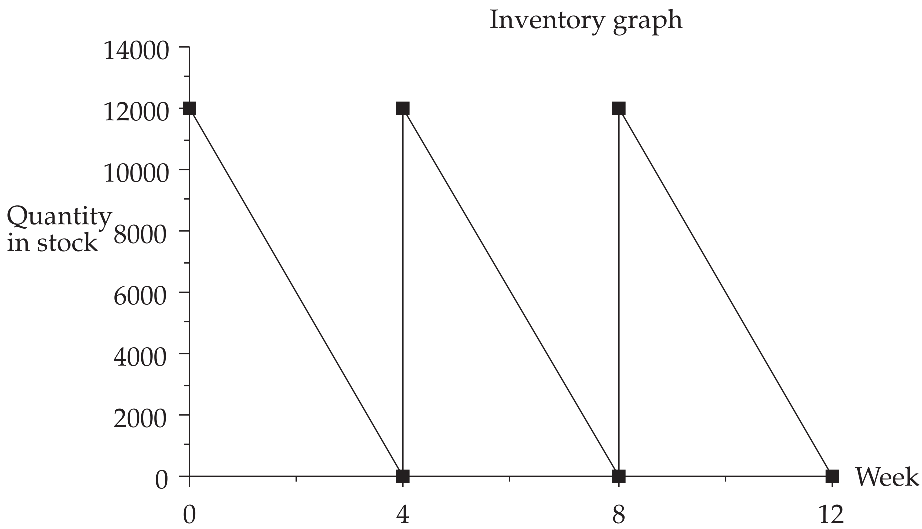
(b) $T = \begin{bmatrix} 0 & 6 & 12 \\ -23 & 12 & 0 \end{bmatrix}$ (c) 2(M*N) + 3[(M+T)*N]

10. (a) $C^*Q = \begin{bmatrix} 60 & 6q \\ 35 & 8q \end{bmatrix}$ which describes the total cost functions for products x and y

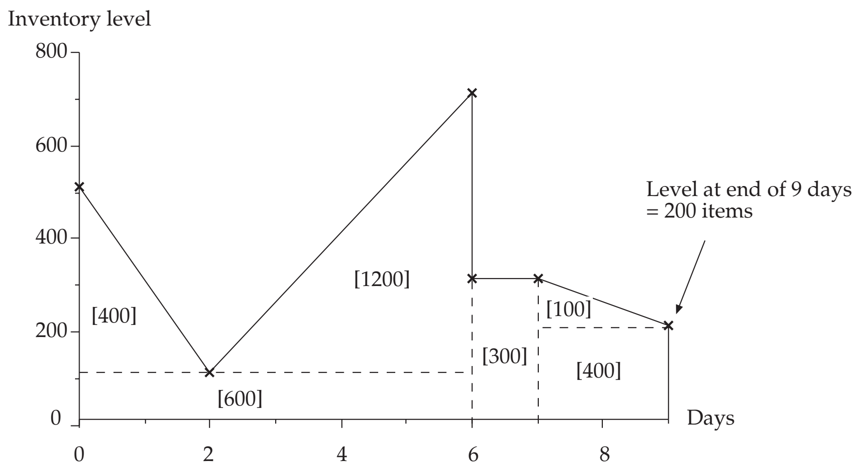
(b) $R = \begin{bmatrix} 0 & 9 \\ 0 & 10 \end{bmatrix}$ (c) $R^*Q - C^*Q = \begin{bmatrix} 3q-60 \\ 2q-35 \end{bmatrix}$ which describes the profit functions for products x and y. (d) q=35, giving the level of production that yields the same total profit (of £15) for each product.

Chapter 37 – Inventory control

1.



2.



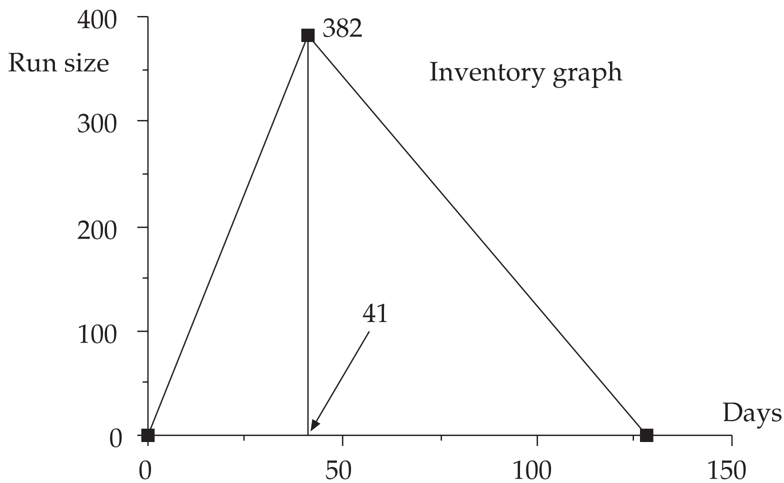
Constituent areas under the curve are shown in square brackets and they total 3000. Thus, the average inventory level over the whole period = 3000/9 = 333.

3. (a) and (b) EOQ = 637; (c) 14 days, 25.9 runs. 4. EOQ = 84852, no. of orders = 7; length of cycle = 51.6 days.

5. Order every 49 (48.6) days. 6. (a) Every 73 days; EOQ = 1000 boxes (b) £187.30 (94% increase in costs).

7. (a) 560 (b) 41 days (c) 128 days (d) 191

(b)



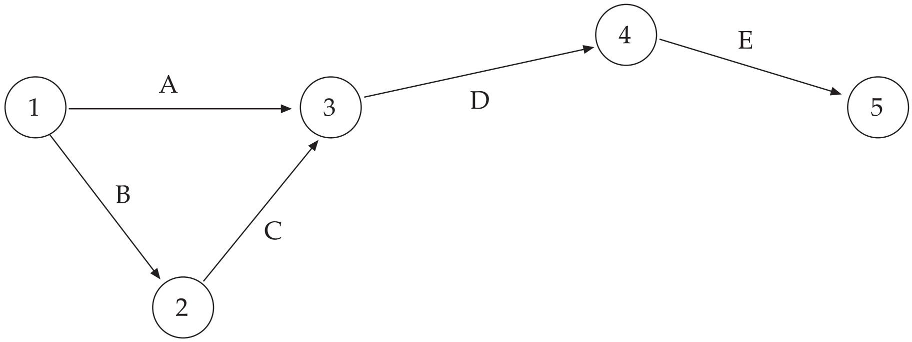
8. EBQ = 3162; every 231 days.

9. Every 3.4 days.

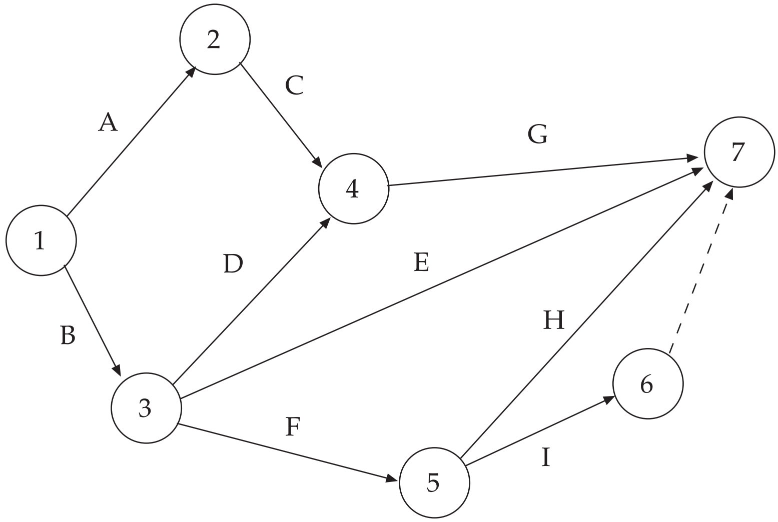
Chapter 38 – Network planning and analysis

1.

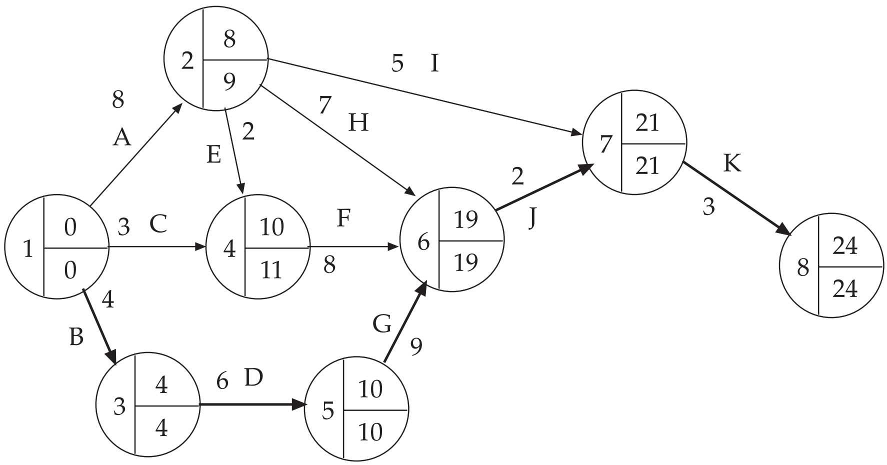
Activity	*Preceding activity*
A,B	–
C	B
D	A,C
E	D



2. a) AIK = 8+5+3 = 16; AHJK = 8+7+2+3 = 20; AEFJK = 8+2+8+2+3 = 23; CFJK = 3+8+2+3 = 16; BDGJK = 4+6+9+2+3 = 24
   b) Total project time = 24. Critical path is BDGJK.
3. a) See diagram on next page
   b) ACG = 11; BDG = 9; BE = 9; BFH = 12; BFI[] = 11
   c) Critical path is BFH with duration time 12.



4.



The critical path (BDGJK) has been *enboldened* and agrees with the result of exercise 2.

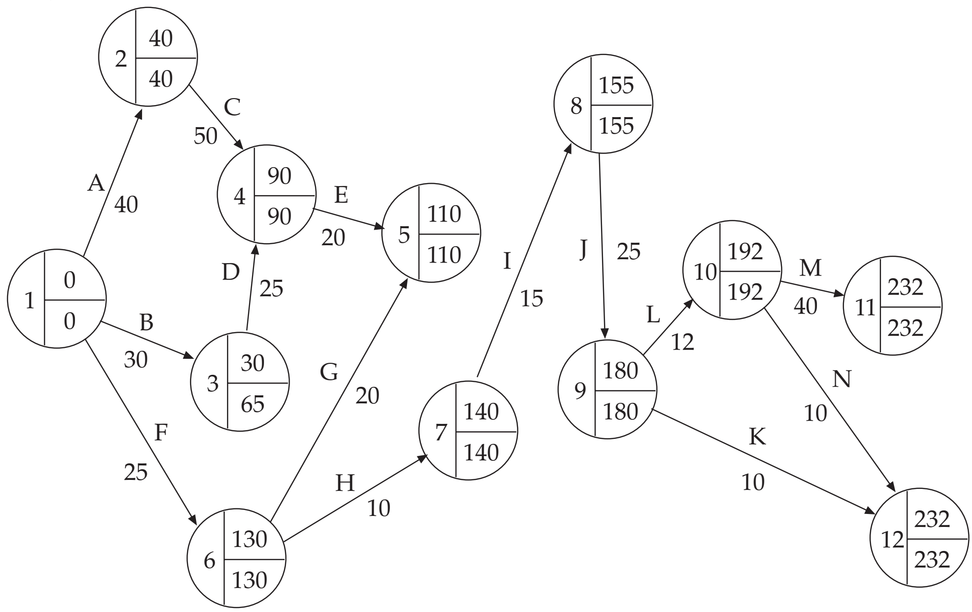
5.

Activity	*Duration*	*Start*		*Finish*		*Floats*		
		E	L	E	L	Tot	Free	Ind
1–2	8	0	1	8	9	1	0	0
1–3	4	0	0	4	4	0	0	0
1–4	3	0	8	3	11	8	7	7
2–4	2	8	9	10	11	1	0	0
2–6	7	8	12	15	19	4	4	3
2–7	5	8	16	13	21	8	8	7
3–5	6	4	4	10	10	0	0	0
4–6	8	10	11	18	19	1	1	0
5–6	9	10	10	19	19	0	0	0
6–7	2	19	19	21	21	0	0	0
7–8	3	21	21	24	24	0	0	0

6. a)

Activity	*Preceding activity*
A,B,F	–
D	A
C	B
J	I
L,K	J
M,N	L
G	E
H	G,F
E	D,C

b)



c)

Activity	*Duration*	*Start* E	*Start* L	*Finish* E	*Finish* L	*Floats* Tot	*Floats* Free	*Floats* Ind
A	40	0	0	40	40	0	0	0
B	30	0	35	30	65	35	0	0
C	50	40	40	90	90	0	0	0
D	25	30	65	55	90	35	35	0
E	20	90	90	110	110	0	0	0
F	25	0	105	25	130	105	105	105
G	20	110	110	130	130	0	0	0
H	10	130	130	140	140	0	0	0
I	15	140	140	155	155	0	0	0
J	25	155	155	180	180	0	0	0
K	10	180	222	190	232	42	42	42
L	12	180	180	192	192	0	0	0
M	40	192	192	232	232	0	0	0
N	10	192	222	202	232	30	30	30

Answers to examination questions

Part 1

Question 1

Simple random sampling. A method of sampling whereby each member of the population has an equal chance of being chosen. Normally, random sampling numbers are used to select individual items from some defined sampling frame.

Stratification. This is a process which splits a population up into as many groups and sub-groups (strata) as are of significance to the investigation. It can be used as a basis for quota sampling, but more often is associated with stratified (random) sampling. Stratified sampling involves splitting the total sample up into the same proportions and groups as that for the population stratification and then separately taking a simple random sample from each group. For example, employees of a company could be split into male/female, full-time/part-time and occupation category.

Quota sampling. A method of non-random sampling which is popular in market research. It uses street interviewers, armed with quotas of people to interview in a range of groups, to collect information from passers-by. For example, obtaining peoples' attitudes regarding the worth of secondary double glazing.

Sample frame. This is a listing of the members of some target population which needs to be used in order to select a random sample. An example of a sampling frame would be a stock list, if a random sample was required from current warehouse stock.

Cluster sampling. This is another non-random method of sampling, used where no sampling frame is in evidence. It consists of selecting (randomly) one or more areas, within which all relevant items or subjects are investigated. For example, a cluster sample could be taken in a large town to interview tobacconists.

Systematic sampling. A quasi-random method of sampling which involves examining or interviewing every n-th member of a population. Very useful method where no sampling frame exists, but population members are physically in evidence and ordered. For example, items coming off a production line. It is virtually as good as random sampling except where the items or members repeat themselves at regular intervals, which could lead to serious bias.

Question 2

(a) A postal questionnaire is a much cheaper and more convenient method of collecting data than the personal interview and often very large samples can be taken. However, much more care must be taken in the design of the questions, since there will be no help to hand if questions seem ambiguous or personal to the respondent. Also the response rate is very low, sometimes less than 20%, but this can sometimes be made larger by free gifts or financial incentives.

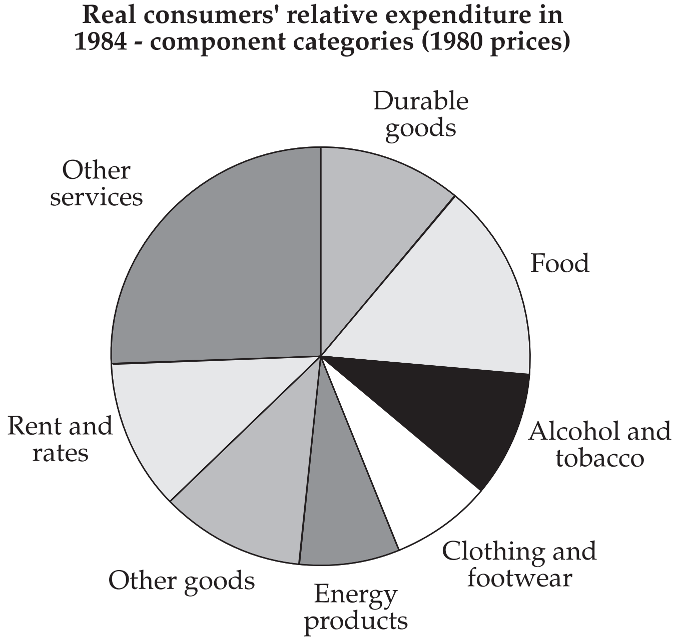
The personal interview has the particular advantage that difficult or ambiguous questions can be explained as well as the fact that an interviewer can make allowances or small adjustments according to the situation. Also, the questionnaire will be filled in as required. Disadvantages of this method include the cost, the fact that large samples cannot generally be undertaken and the training of interviewers.

(b) Simple random sampling has the particular advantage that the method of selection (normally through the use of random sampling numbers) is free from bias. That is, each member of the population has an equal chance of being chosen as part of the sample. However, it cannot be guaranteed that the sample itself is truly representative of the population. For example, if a human population being sampled comprised 48% males, it is unlikely that the sample would reflect this percentage exactly.

Quota sampling is not a random sampling method and thus is generally at a disadvantage with regard to obtaining information that can claim to be representative. However, if the population has been stratified reasonably well, the street interviewer is experienced and conscientious and the questioning sites have been well thought out, it could be argued that, in certain localised situations, a quota sample could be very representative. For example, to gauge peoples opinions of a new shopping centre or to find out the views of theatregoers about a particular theatre.

Question 3

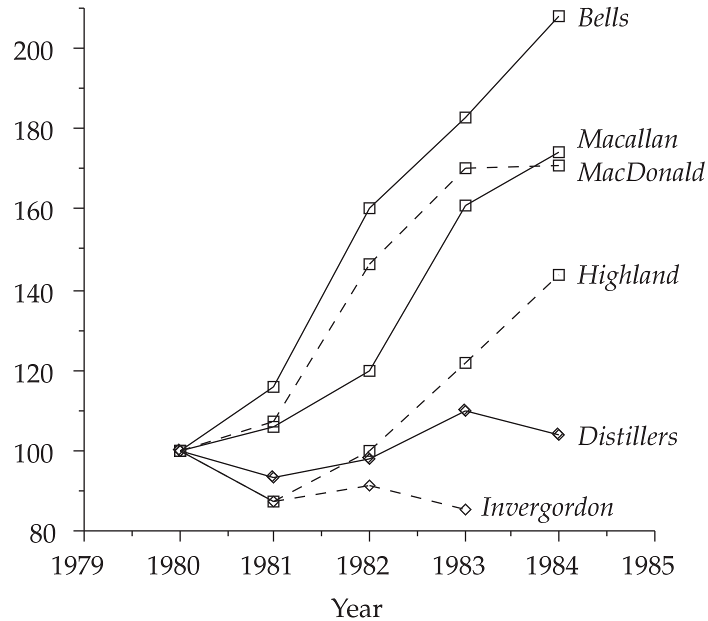
(a) (i) See pie chart.



(ii) Other goods: books, toys, toiletries, transport. Other services: insurance, recreation, entertainment, (private) dental/health care.

(b) See line diagram.

Comparative profit before tax (1980=100) of six Scotch whisky companies



Question 4

(a) i. An absolute error is the difference between an estimated value and its true value. In most cases, only a maximum absolute error will be able to be calculated. For example, if a company's yearly profit was quoted as £252,000 (to the nearest £1000), the maximum absolute error would be £500.

ii. A relative error is an absolute error expressed as a percentage of the given estimated value. Thus in the example above, the maximum relative error in the company's yearly profit is:

$$\frac{500}{252{,}000} \times 100\% = 0.2\%$$

iii. A compensating error is an error that is made when 'fair'rounding has been carried out. For example, the numbers of people employed in each of a number of factories might well be rounded fairly, to the nearest 1000, say. When numbers, subject to compensating errors, are added, the total relative error should be approximately zero.

iv. Biased errors are made if rounding is always carried out in one direction. For example, when people's ages are quoted, they are normally rounded *down* to the lowest year. The error in the sum of numbers that are subject to biased errors is relatively high.

(b)

	Minimum	Estimate	Maximum
Time	145 hours	150 hours	155 hours
Wage rate	£4/hr	£4/hr	£4.40/hr
Labour cost	£580	£600	£682
Material cost	£2550	£2600	£2650
Total cost	£3130	£3200	£3332
Quote	£4000	£4000	£4000
PROFIT	£668	£800	£870

Question 5

(a) Smallest value = 347; largest value = 469. Thus, range = 122.
Since five classes are required, a class width of 122÷5 = 24.4, adjusted up to 25, seems appropriate.

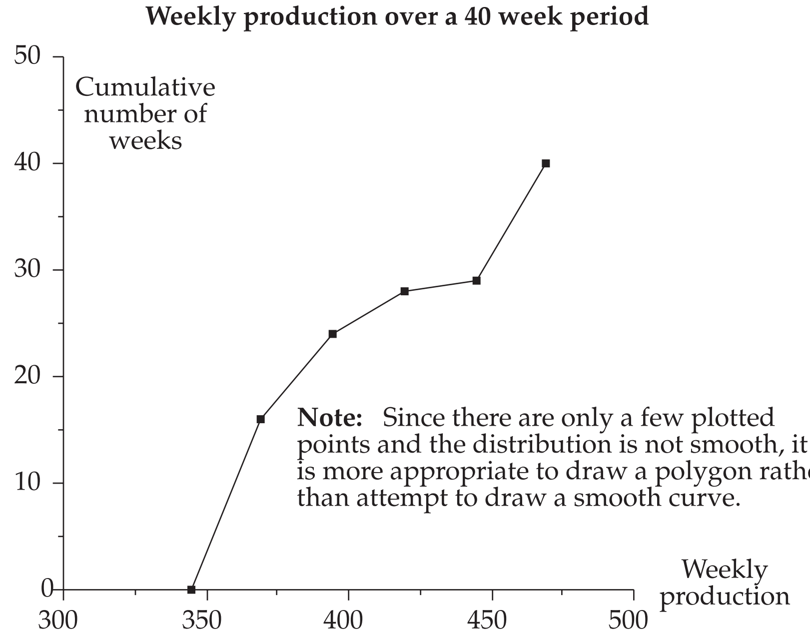
Weekly production	Tally	Number of weeks
345 to 369	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16
370 to 394	~~IIII~~ III	8
395 to 419	IIII	4
420 to 444	I	1
445 to 469	~~IIII~~ ~~IIII~~ I	11
	Total	40

(b) To construct the ogive, *cumulative frequency* needs to be plotted against *class upper bounds*.

Weekly production (upper bound)	Cumulative number of weeks
369.5	16
394.5	24
419.5	28
444.5	29
469.5	40

The ogive is shown in the figure following.

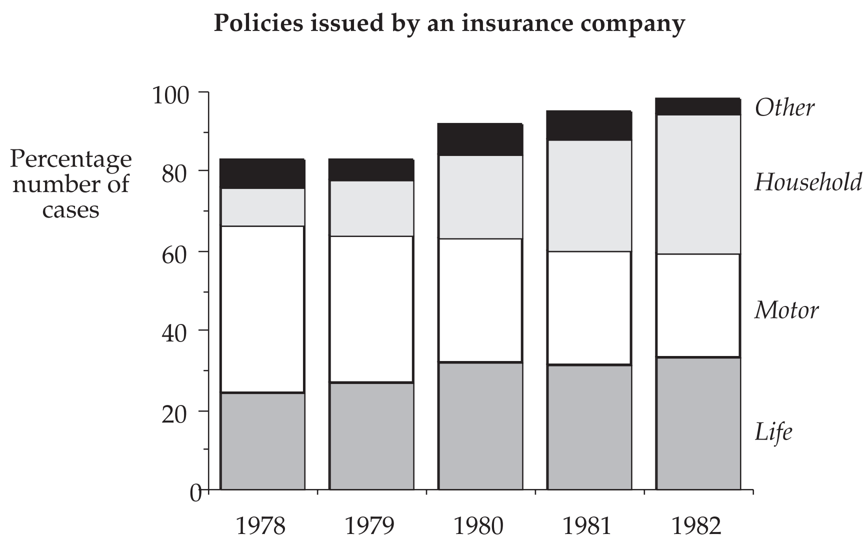
Figure 1



Question 6

(a) Although generally a component time series is best represented by a (cumulative) line diagram, in this case, since there are so few time points, a component bar chart has more impact. The chart is drawn in Figure 2.

Figure 2



(b) Component bar charts enable comparisons between components across the years to be made easily, showing also yearly totals. The main disadvantage is the fact that actual values cannot easily be determined.

(c) Overall, there has been a steady increase in the number of policies issued each year. Household policies have shown a steady increase over the five year period at the expense of Motor, which have steadily decreased. Life has shown a very small increase over the period except for a small dip in 1981. Other policies have remained fairly steady, fluctuating only slightly around 6,000.

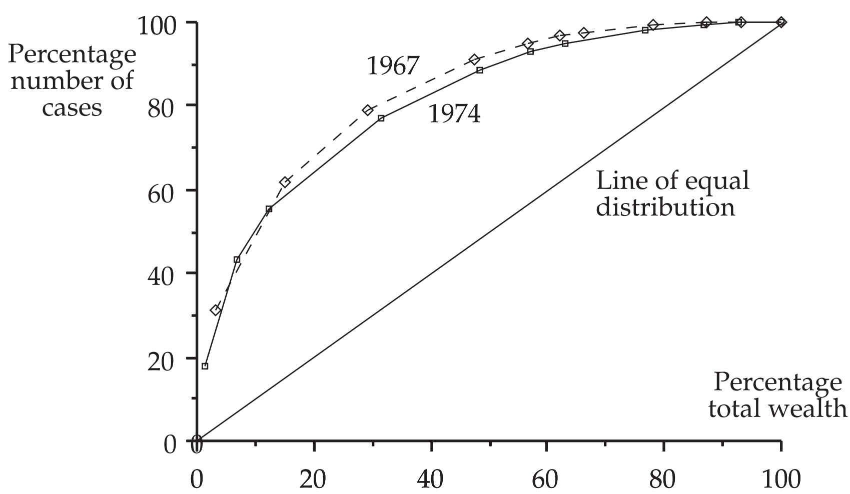
The information given concerns only numbers of new policies actually issued. No indication is given of premium values, cancellations or claims, therefore nothing can be said about the financial progress of the company.

Question 7

The standard calculations for the plotting of the two Lorenz curves are shown in Table 1 and the two corresponding Lorenz curves are plotted in Figure 3.

Figure 3

Identified personal wealth in the UK for 1967 and 1974



It can be seen from Figure 3 that the distribution of wealth in both years is similar, showing little change over the seven year period. There has been a very small redistribution towards equality, but this is not marked. In both years, the figures show that the least wealthy 50% of the population own only 10% of total wealth. However, 50% of all wealth was owned by the wealthiest 8% in 1967, while in 1974 it was shared between the wealthiest 10%.

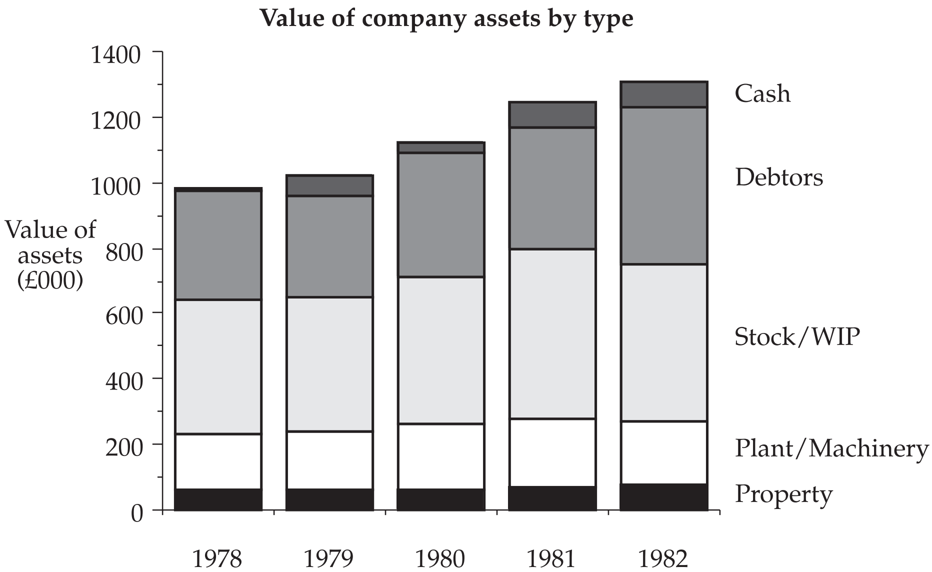
Table 1

Range of wealth (£000)	Number of cases		Total wealth		Number of cases		Total wealth	
	%	cum %	%	cum %	%	cum %	%	cum %
0 to 1	31.2	31.2	3.4	3.4	18.1	18.1	1.3	1.3
1 to 3	30.5	61.7	11.7	15.1	25.4	43.5	5.5	6.8
3 to 5	17.1	78.8	13.9	29.0	11.8	55.3	5.5	12.3
5 to 10	12.6	91.4	18.3	47.3	21.9	77.2	19.3	31.6
10 to 15	3.6	95.0	9.1	56.4	11.5	88.7	16.9	48.5
15 to 20	1.6	96.6	5.7	62.1	4.0	92.7	8.5	57.0
20 to 25	0.9	97.5	4.1	66.2	2.2	94.9	6.1	63.1
25 to 50	1.6	99.1	11.8	78.0	3.4	98.3	13.8	76.9
50 to 100	0.6	99.7	9.0	87.0	1.2	99.5	9.8	86.7
100 to 200	0.2	99.9	6.1	93.1	0.4	99.9	5.8	92.5
over 200	0.1	100	6.9	100	0.1	100	7.5	100

Question 8

(i) The component bar chart for the given data is drawn in Figure 4.

Figure 4



(ii) Overall, the total value of the given assets has increased steadily from just under £1m in 1978 to £1.3m in 1982. The most significant increase has been the debtors component, which has caught up with the stock and work-in-progress component, even though the latter has also increased. The property component shows very small increases, while plant and machinery shows small increases in the first four years and a decrease in the fifth year. Although the cash component has fluctuated over the five year period, it has shown an increase and is now comparable with property.

Question 9

(a) (i) *Pictogram*. A representation that is easy to understand for a non-sophisticated audience. However, it cannot represent data accurately or be used for any further statistical work.

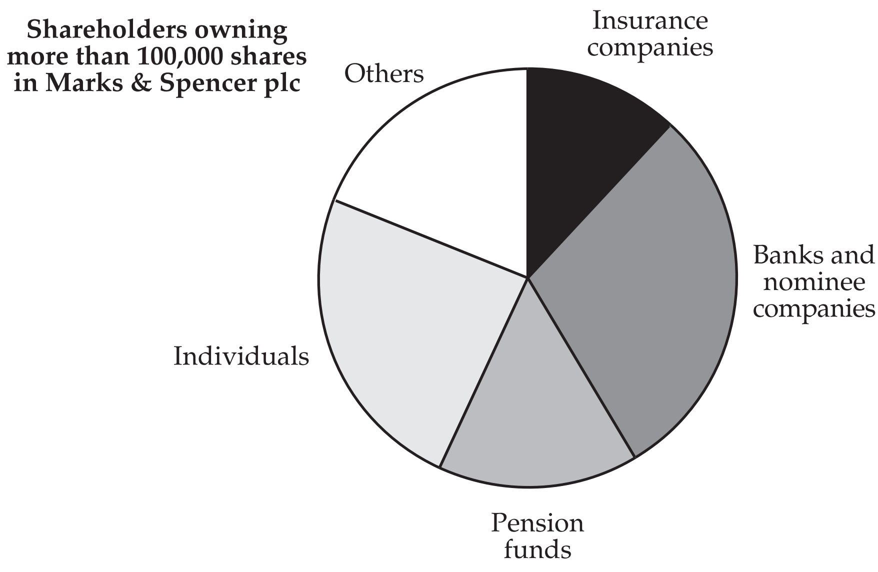
(ii) *Simple bar chart*. One of the most common forms of representing data which can be used for time series or qualitative frequency distributions. It is easy to understand and can represent data accurately. However, data values are not easily determined.

(iii) *Pie chart*. A type of chart which can have a lot of impact. Used mainly where the classes need to be compared in relative terms. However, they involve fairly technical calculations.

(iv) *Simple line diagram*. The simplest and most popular form of representing time series. They are easy to understand and represent data accurately. However, data values are not easily determined.

(b) A pie chart is one of the charts that could be drawn for the given data and is shown at Figure 5. Note however that a simple bar chart could equally well represent the data.

Figure 5



Question 10

The company can make and sell 10,000 ± 2,000 units in the year

The selling price will lie in the range £50 ± £5 per unit

Thus the maximum revenue is 12,000 × £55 = £660,000

The minimum revenue is 8,000 × £45 = £360,000

The estimated revenue is 10,000 × £50 = £500,000

The maximum error from the estimated revenue is £660,000 – £500,000 = £160,000

and relative error = $\frac{150,000}{500,000} \times 100\% = 32\%$

The ranges of the various costs are:

	min		max	
materials	£147,000	to	£153,000	
wages	£95,000	to	£105,000	
marketing	£45,000	to	£55,000	
miscellaneous	£45,000	to	£55,000	
Total:	£332,000	to	£368,000	est = £350,000

Maximum error from the estimated costs = £18,000

and relative error = $\frac{80{,}000}{350{,}000} \times 100\% = 5.1\%$

Maximum contribution = £660,000 – £332,000 = £328,000

Minimum contribution = £360,000 – £368,000 = –£8,000

the estimated contribution = £150,000

The maximum error from the estimated contribution is £328,000 – £150,000 = £178,000

Which gives the relative error $\frac{178{,}000}{150{,}000} \times 100\ \% = 118.7\%$

The maximum contribution of £328,000 arises when 12,000 units are made and sold

Therefore contribution/unit = $\frac{£328{,}000}{12{,}000}$ = £27.33/unit

The minimum contribution of – £8,000 arises when 8,000 units are made and sold

Therefore contribution/unit = $\frac{-£8{,}000}{8{,}000}$ = – £1

The estimated contribution /unit = £15

The maximum error from the estimated contribution/unit is £15 – (–£1) = £16

Therefore, relative error = relative error as $\frac{16}{15} \times 100\% = 106.7\%$.

Part 2

Question 1

True limits							
Lower	Upper	Mid-point	f	fx	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
0	5	2.5	39	97.5	–34.16	1,166.91	45,509.31
5	15	10	91	910	–26.66	710.75	64,678.75
15	30	22.5	122	2,745	–14.16	200.51	24,462.22
30	45	37.5	99	3,712.5	0.84	71	70.29
45	65	55	130	7,150	18.33	335.99	43,678.70
65	75	70	50	3,500	33.34	1,111.56	55,578.00
75	95	85	28	2,380	48.34	2,336.76	65,429.28
Total			*559*	*20,495*			*299,406.55*

The upper class limit of the final class is such that the class width is double that of the preceding class.

Mean, $\bar{x}$ = 20,495/559 = 36.66

$$\text{Standard deviation} = \sqrt{\frac{299{,}406.55}{559}} = 23.14$$

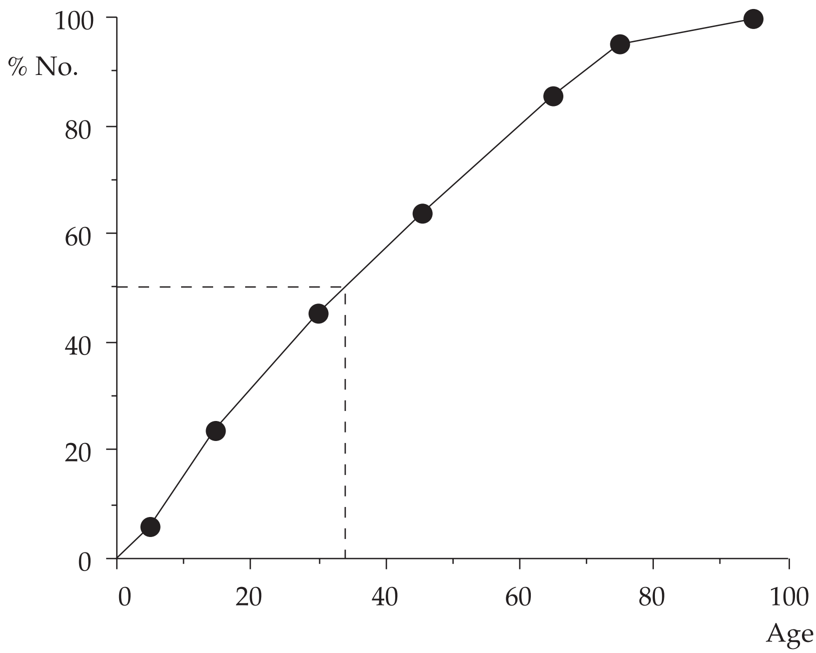
For the histogram plot, since the class widths are uneven, we would need to scale the frequencies to plot heights for all those bars that are not standard width. This is due to the fact that the area of histogram bars (NOT height – except for classes that are all the same width) should represent frequency. Since most of the classes are of different widths, the calculations will be impractical and *not advisable in an examination*.

[AUTHOR NOTE: Clearly the examiner has made an error. This can be verified by reference to the suggested solution published by the examining board (ACCA) which shows the heights of bars representing frequencies. The correct histogram is too tedious to calculate and draw and thus is not represented here!]

Lower x	Upper x	f	%	Cumulative %
0	5	39	6.98	6.98
5	15	91	16.28	23.26
15	30	122	21.82	45.08
30	45	99	17.71	62.79
45	65	130	23.26	86.05
65	75	50	8.94	94.99
75	95	28	5.01	100

Median = 34. See following graph.

Age distribution



Question 2

(a) (i) Only 100 – 36.4 = 63.6% received training. Hence, required percentage

$$= \frac{36.2}{63.6} \times 100 = 56.9\%$$

(ii) Similarly, $\frac{7.2}{100 - 48.2}$ = 13.9% received training.

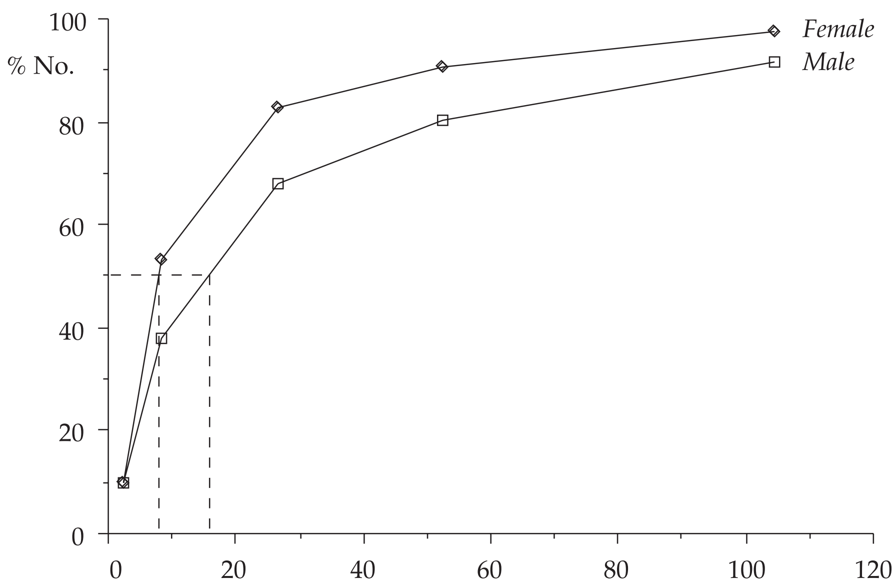
(b) *Non-apprentices receiving training*:

	Male %	Male cum %	Female %	Female cum %
1 to 2	9.9	9.9	9.8	9.8
3 to 8	28.1	38.1	43.3	53.1
9 to 26	30.0	68.1	29.5	82.6
27 to 52	12.3	80.4	7.9	90.6
53 to 104	11.1	91.5	6.7	97.3
105 or more	8.5	100.0	2.7	100.0

Note: Each male percent in the above is calculated using the given table % as a percentage of 100 – 57.7.

For example, $9.9 = \frac{4.2}{100 - 57.7} \times 100$. Similarly for females.

Length of training of non-apprentices



From the graph, the median for male non-apprentices is 16 weeks and the median for female non-apprentices is 8 wks.

(c) All apprentices receive at least 1 year's training. Non-apprentices receive 10 to 11 weeks training on average. Males receive more training than females in general. Although a greater proportion of all females receive no training at all, more female non-apprentices receive *some* training than their male counterparts. A much greater proportion of males are apprenticed (37%) than females (8%).

Question 3

Group	Mid-point x	f	fx	fx^2
30 but less than 35	32.5	17	552.5	17956
35 but less than 40	37.5	24	900.0	33750
40 but less than 45	42.5	19	807.5	34319
45 but less than 50	47.5	28	1330.0	63175
50 but less than 55	52.5	19	997.5	52369
55 but less than 60	57.5	13	747.5	42981
		120	5335.0	244550

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{5335}{120} = 44.5 \text{ milliseconds}$$

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{244550}{559120} - \left(\frac{5335}{120}\right)^2} = 7.8 \text{ milliseconds}$$

Since the data are grouped, and thus the original access times are not known, both the measures above are estimates.

Question 4

(a) Smallest value = 3; largest value = 33; range = 30.

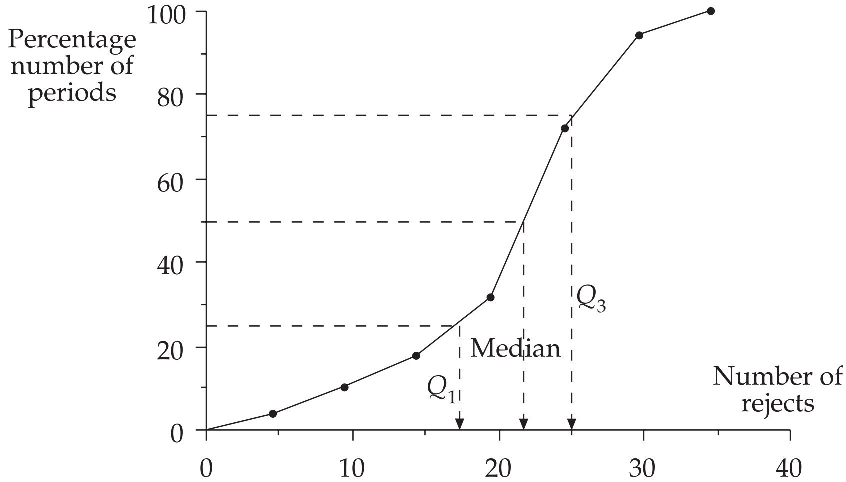
Seven classes will each have a class width of 30÷7 = 5 (approx). The formation of a cumulative frequency table is shown at Table 2.

Table 1

Number of rejects		Number of periods (f)	Cum f (F)	F%
0 to 4	II	2	2	4
5 to 9	III	3	5	10
10 to 14	IIII	4	9	18
15 to 19	~~IIII~~ II	7	16	32
20 to 24	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~	20	36	72
25 to 29	~~IIII~~ ~~IIII~~ I	11	47	94
30 to 34	III	3	50	100

(b) Because the distribution is skewed, the median and quartile deviation are appropriate measures to describe the distribution.

Figure 1



From the graph at Figure 2: Q_1 = 17.5; median = 21.5; Q_3 = 25.

Therefore, quartile deviation $= \dfrac{Q_3 - Q_1}{2} = \dfrac{150 + 120}{2} = 3.8$

(c) The median of 21.5 describes the average number of rejects in each five minute period = 260/hr (approx). The quartile deviation measures the variability in the number of rejects from one five minute period to the next. In particular, we expect 50% of rejects to lie within 21.5 ± 3.8 in one five minute period.

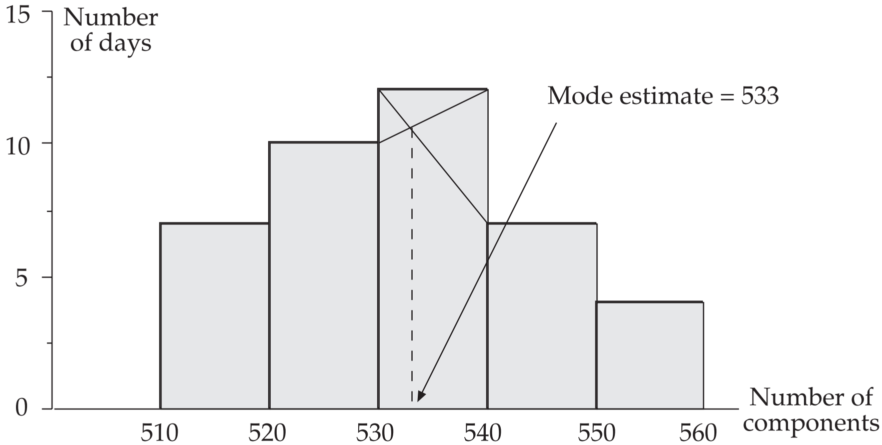
Question 5

(a) Smallest value = 510; largest value = 555; range = 45. For 5 classes, each class should have width of 45÷5 = 9 (but 10 is better!)

Number of components		*f*
510-519	~~IIII~~ II	7
520-529	~~IIII~~ ~~IIII~~	10
530-539	~~IIII~~ ~~IIII~~ II	12
540-549	~~IIII~~ II	7
550-559	IIII	4

(b) (c) Figure 3 shows the histogram and, from it, the calculation of the mode.

Figure 2



From the histogram, mode = 533.

(d) (e)

x	f	fx	fx^2
514.5	7	3601.5	1852971.7
524.5	10	5245	2751002.5
534.5	12	6414	3428283
544.5	7	3811.5	2075361.7
554.5	4	2218	1229881
	40	21290	11337499

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{21290}{40} = 532.25$$

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{11337499}{40} - \left(\frac{21290}{40}\right)^2} = 12.14 \text{ (2D)}$$

(f) Mode = 533; mean = 532.25; sd = 12.14. Mode>mean implies *slight* left skew, which can just be made out from the frequency distribution.

Question 6

Income (£)	$f\%$	$F\%$	
Up to 30,000	5	5	
30,000-34,999	2	7	
35,000-35,999	3	10	
40,000-44,999	5	15	
45,000-49,999	10	25	
50,000-59,999	15	40	← 1st F value to exceed 25
60,000-69,999	18	58	← 1st F value to exceed 5(
70,000-99,999	21	79	← 1st F value to exceed 75
100,000-149,999	17	96	
150,000 and over	4	100	

Since the data is skewed and the first and last classes are open-ended, the median and quartile deviation are the most suitable measures of location and dispersion. The interpolation formula is used below to calculate the measures.

$$Q_1 = 49999.5 + \frac{25-25}{15} \times 10000 = 49{,}999.5\ (50{,}000)$$

$$\text{Median} = 59999.5 + \frac{50-40}{18} \times 10000 = 65{,}555.1\ (65{,}555)$$

$$Q_3 = 69999.5 + \frac{75-58}{21} \times 30000 = 94{,}285.1\ (94{,}285)$$

$$\text{Quartile deviation} = \frac{94{,}285 - 50{,}000}{2} = 22{,}142.$$

Question 7

(a) Average rates of increase are usually found using the geometric mean.
For example:

Year	1	2	3	4
Rate of increase	2.3%	3.8%	1.9%	4.2%

The appropriate multipliers are 1.023, 1.038, 1.019 and 1.042

Thus average multiplier = Geometric mean

$$= \sqrt{1.023 \times 1.038 \times 1.019 \times 1.042}$$

Therefore, average rate of increase = 3.05%

(b) In a skewed distribution, particularly where only a few values are contained at just one end, the median is the appropriate average to use since it largely ignores extremes and it would be giving the information that 50% of all values are less, and 50% more, than the median value.

(c) Since the speeds need to be averaged *over the same distance*, the harmonic mean is the appropriate average.

$$\text{hm} = \frac{2}{\frac{1}{30}+\frac{1}{60}} = 40 \text{ mph}$$

(However, if the speeds needed to be averaged *over the same time*, the arithmetic mean would be used, giving am = $\frac{30+60}{2}$ = 45 mph.)

(d) An average would not be appropriate at all here, since clearly some of the ships would not be able to pass under a bridge built to this height. The height necessary needs to be (at least) the largest value in the distribution.

(e) A weighted mean would be appropriate here. If there were n_1 skilled and n_2 unskilled workers, the income for each of the workers could be calculated as:

$$\frac{4500 \times n_1 + 3500 \times n_2}{n_1 + n_2}$$

(f) A simple mean is all that is required.

i.e. mean amount = $\dfrac{\text{Total profits to be allocated}}{\text{Number of employees}}$

Question 8

The table showing calculations is given below.

	Mid-point (x)	(f)	(fx)	(fx^2)
40 to 60	50	5	250	12500
60 to 80	70	7	490	34300
80 to 100	90	7	630	56700
100 to 120	110	18	1980	217800
120 to 140	130	23	2990	388700
140 to 160	150	14	2100	315000
160 to 180	170	10	1700	289000
180 to 220	200	16	3200	640000
		100	13340	1954000

(i) $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{13340}{100} = 133.4$

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{1954000}{100} - \left(\frac{13340}{100}\right)^2} = 41.77 \text{ (2D)}$$

(ii) $\text{cv}(1) = \dfrac{41.77}{133.4} \times 100 = 31.3\%$; $\text{cv}(2) = \dfrac{29.33}{88.0} \times 100 = 33.3\%$.

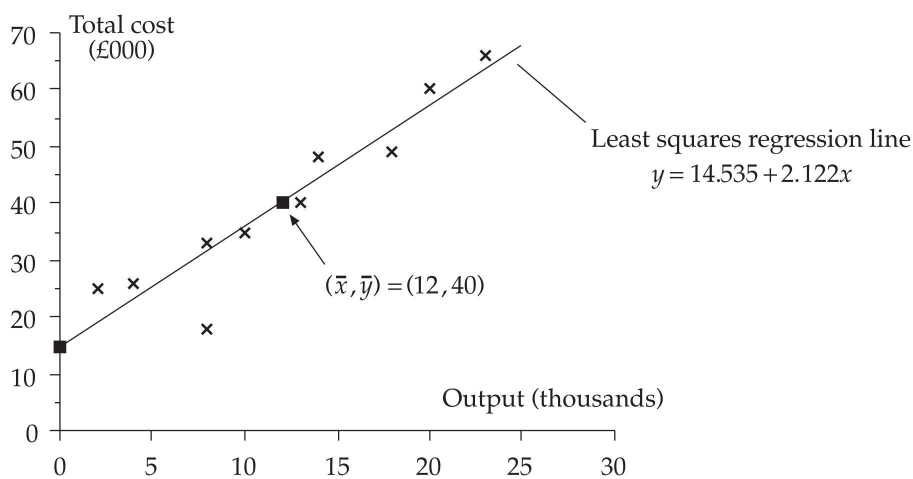
(iii) Distribution 2 is relatively more variable

Part 3

Question 1

(a)

Output and cost of standard size boxes



(b) Week 8's figures of 8000 output at a total cost of £18000 are distinctly out of line with the rest of the data. This is clearly due to special circumstances, perhaps a cheap off-loading of old stock.

(c) Any regression line fitted to a set of bivariate data must pass through the mean point $(\bar{x}, \bar{y})$.

In this case, $\bar{x} = \frac{120}{10} = 12$ and $\bar{y} = \frac{400}{10} = 40$.

(d) Let the regression line be in the form: $y = a + bx$. Using the least squares technique, we have:

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{10(5704) - 120(400)}{10(1866) - (120)^2} = \frac{9040}{4260} = 2.122 \text{ (3D)}$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n} = 40 - (2.122)12 = 14.535 \text{ (3D)}$$

i.e. least squares line of y (total cost) on x (output) is $y = 14.535 + 2.122x$

For the graph plot, y-intercept is 14.535 and the line must pass through the point (12,40) from part (c) above.

(e) The fixed costs of the factory is just the value of the y-intercept point of the regression line = 14.536 or £14536.

(f) If 25000 standard boxes are produced, then the regression line can be used to estimate the total costs as follows:

Estimated total costs = 14.535 + (2.122)(25) in £000 = £67585.

Question 2

(a) See the diagram below. Since both sets of data are close to their respective regression lines, correlation is quite good (and positive). The average turnover for multiples is higher than that for co-operatives, as evidenced by the higher figures, and, since the gradient of the multiple line is larger, multiples have also the higher marginal turnover.

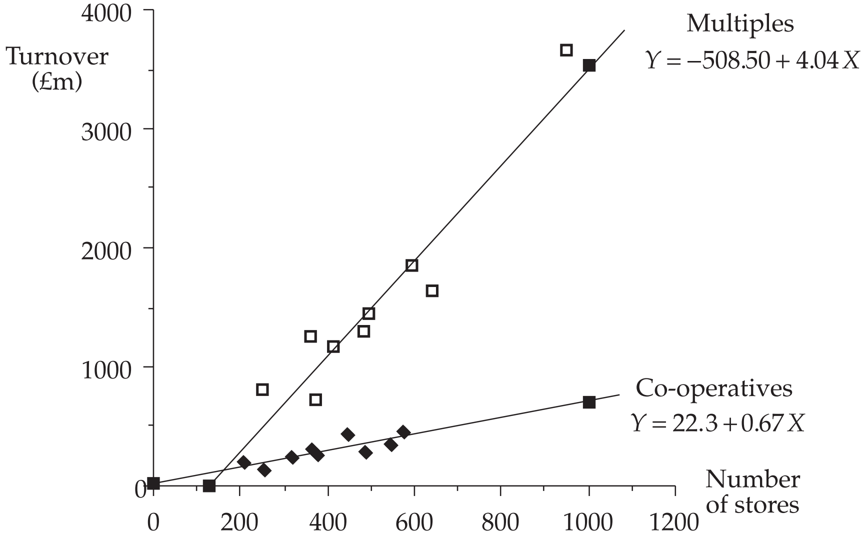
(b) Putting X=500 into both regression lines gives:

multiples: $Y = -508.5 + (4.04)(500) = 1511.5$ i.e. a turnover of £1501m.

co-operatives: $Y = 22.73 + (0.67)(500) = 357.73$ i.e. a turnover of £350m.

Since correlation is high and both estimates have been interpolated, a good degree of accuracy might be expected.

(c) As mentioned in (a), the marginal turnover for multiples is higher than for co-operatives. Specifically, for multiples between 253 and 952 stores, each extra store generates a turnover of £4.04m.; for co-operatives between 210 and 575 stores, each extra store generates a turnover of £0.67m.



Question 3

(a) This statement is correct. Correlation does not attempt to measure the cause and effect that may exist between two variables, only the *strength of the mathematical relationship*. However, if a causal relationship exists between two variables, there should be a fairly high degree of correlation present.

Example 1: x = Milk consumption; y = Number of violent crimes. Clearly there will be high correlation due to higher population, but obviously no causation!

Example 2: x = Distance travelled by salesman; y = Number of sales made. Here a causal relationship is very probable with a resultant high correlation coefficient.

(b) Table for calculations:

Colour TV licences (millions) x	Cinema admissions (millions) y	xy	x^2	y^2
5.0	134	670.0	25.0	17956
6.8	138	938.4	46.24	19044
8.3	116	962.8	68.89	13456
9.6	104	998.4	92.16	10816
10.7	103	1102.1	114.49	10609
12.0	126	1512.0	144.0	15876
12.7	112	1422.4	161.29	12544
12.9	96	1238.4	166.41	9216
78.0	929	8844.5	818.48	109517

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left(n\sum x^2 - \left(\sum x\right)^2\right)\left(n\sum y^2 - \left(\sum y\right)^2\right)}}$$

$$= \frac{(8)(8844.5)-(78)(929)}{\sqrt{\left((8)(818.48)-78^2\right)}\sqrt{\left((8)(109517)-929^2\right)}}$$

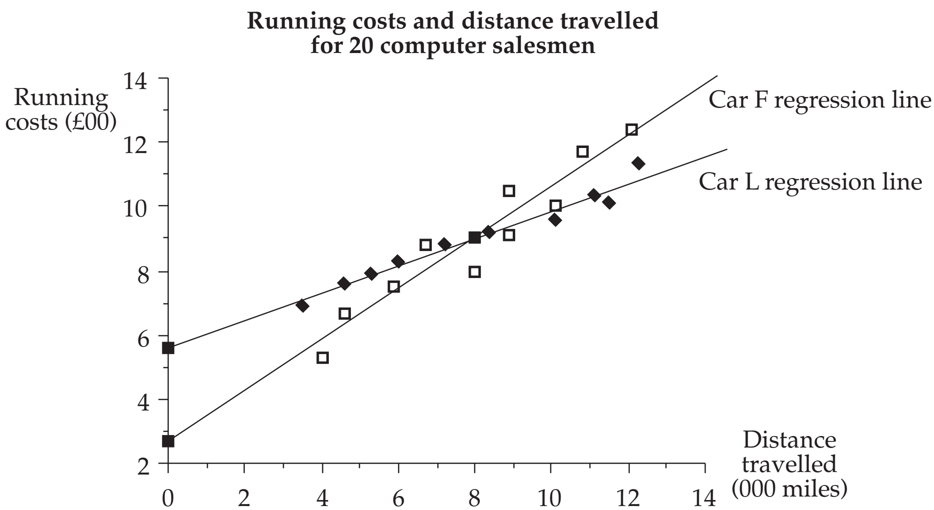
$$= \frac{-1706}{(21.537)(114.433)} = -0.692$$

A moderately high degree of negative correlation, showing that as the number of colour licences increases so the number of cinema admissions decreases.

A causal relationship seems reasonable here, and with $r^2 = 0.48$ (2D), this demonstrates that approximately 50% of the variation in cinema attendances is explained by variations in the number of colour licences.

Question 4

(a) See the figure.



For the regression line plots in the figure,

Car F: intercept on y-axis is 2.65 and line must pass through (8,9).

Car L: intercept on y-axis is 5.585 and line must pass through (8,9).

(b) Car F: 2.65 is the initial (or fixed) running costs (£00) and 0.794 is the extra cost (£00) for each further one thousand miles travelled.

Car L: 5.585 is the initial cost and 0.427 is the extra cost for each further one thousand miles travelled.

(c) It is necessary to minimise the *average* cost per car for the two different types, taking into account the new average distance travelled = 1.5 x 8 = 12 (000 miles).

For type F: Average cost = 2.65 + (0.794)12 = 12.18 = £1218.

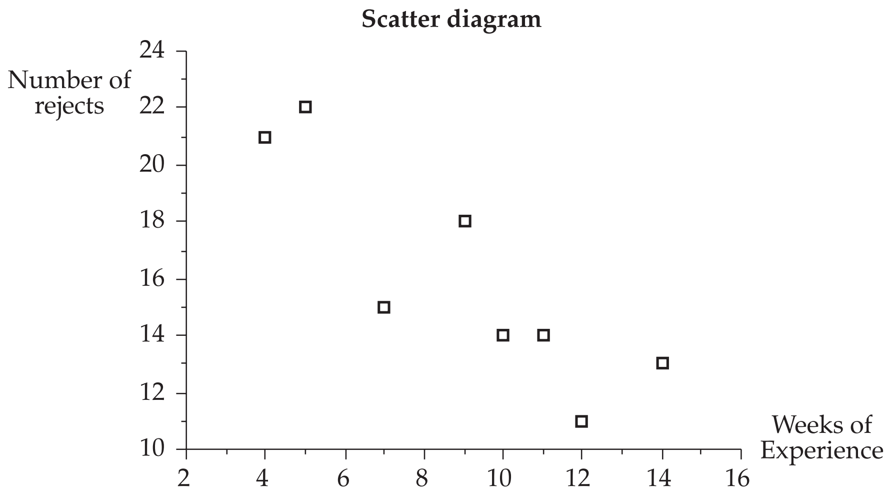
For type L: Average cost = 5.585 + (0.427)12 = 10.71 = £1071.

Therefore car L is cheaper on average.

(d) Using car L, the average cost for one car (with 10% extra costs) is given by (1.1)(5.585 + (0.427)12) = 11.7799 = £1177.99. Thus, expected total running costs for 5 cars is 5 × £1177.99 = £5889.95.

Question 5

(a)



(b) For the product moment correlation coefficient:

$$\bar{x} = \frac{72}{8} = 9 \text{ and } \bar{y} = \frac{128}{8} = 16$$

$$r = \frac{1069 - 8(9)(16)}{\sqrt{\left(732 - 8(9)^2\right)}\sqrt{\left(2156 - 8(16)^2\right)}} = \frac{-83}{(9.165)(10.392)} = -0.87 \text{ (2D)}$$

The result shows high negative correlation and, since a causal relationship seems appropriate, this can be interpreted as the more experience an employee has in wiring components, the fewer the number of rejects to be expected.

c) Assuming a least squares line of the form $y = a + bx$, a and b are calculated as follows:

$$b = \frac{1069 - 8(9)(16)}{4\times3\times2\times1} = \frac{-83}{84} = -0.988 \text{ (3D) and } a = 16 - (-0.988)\times 9 = 24.892 \text{ (3D)}$$

The least squares regression line of y (rejects) on x (experience) is thus:

$y = 24.892 - 0.988x$.

After one week of experience (x=1), the expected number of rejects is given by:

$y = 24.892 - 0.988(1) = 23.9$(1D) or 24 (to nearest whole number).

Question 6

(i) Table for calculations:

Value	rank	Value	rank	d^2	Value	rank	d^2
15	3	13	2	1	16	2	1
19	5	25	5	0	19	3.5	2.25
30	7	23	4	9	26	6	1
12	2	26	6	16	14	1	1
58	8	48	8	0	65	8	0
10	1	15	3	4	19	3.5	6.25
23	6	28	7	1	27	7	1
17	4	10	1	9	22	5	1
				40			12.5

(1) Coefficient for actual and forecast 1:

$$r' = 1 - \frac{6(40)}{8(63)} = 0.52$$

(2) Coefficient for actual and forecast 2:

$$r' = 1 - \frac{6(12.5)}{8(63)} = 0.85$$

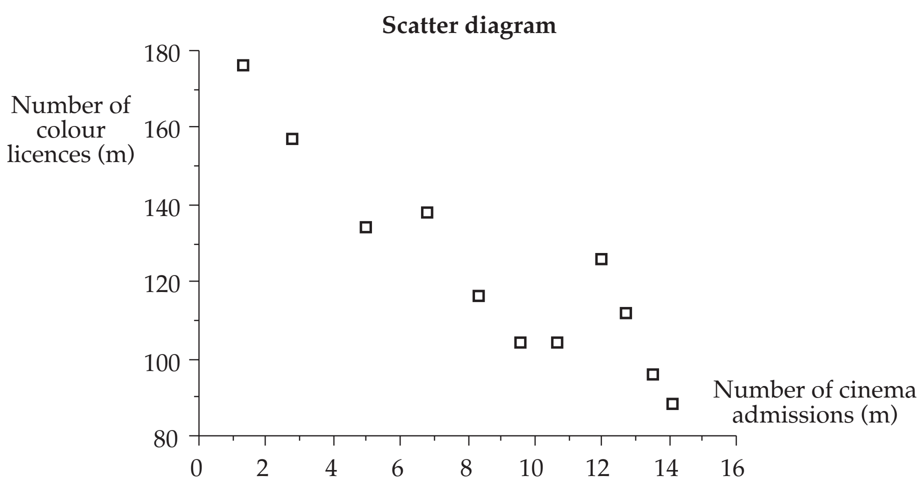
(ii) Clearly, forecasting method 2 is superior.

Question 7

(a) Briefly, regression describes the mathematical (linear) relationship between two variables while correlation describes the strength of this linear relationship.

(b) (i) See the figure on the following page.

The figure clearly shows that as the number of colour licences increases, so the number of cinema attendances decreases.



(ii)

Number of TV licences (m)	rank	Number of cinema admissions (m)	rank	d^2
1.3	1	176	11	100
2.8	2	157	10	64
5.0	3	134	8	25
6.8	4	138	9	25
8.3	5	116	6	1
9.6	6	104	3.5	6.25
10.7	7	104	3.5	12.25
12.0	8	126	7	1
12.7	9	112	5	16
13.5	10	96	2	64
14.1	11	88	1	100
				414.5

Rank correlation coefficient: $r' = 1 - \dfrac{6(414.5)}{11(120)}$

$$= -0.88$$

The above coefficient is showing strong inverse (or negative) correlation and, since there is every reason to believe that there is a causal relationship here, the hypothesis seems reasonable.

Question 8

(a) As the regression equation of profit on sales is required, put profit = y and sales = x

The equation is:

$$y = a + bx,$$

where: $b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$

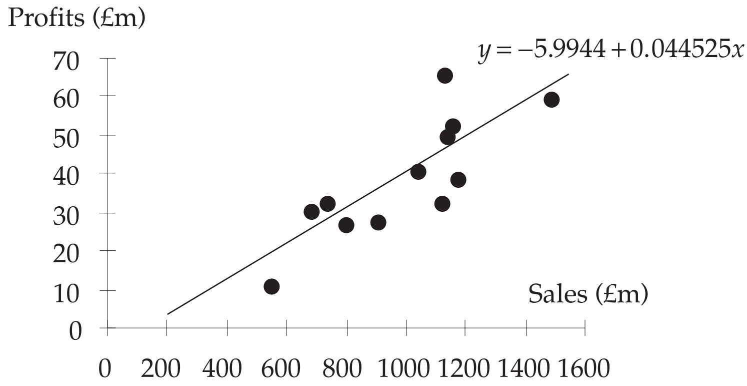
$$= \frac{12 \times 498,912.2 - 11,944 \times 462.1}{12 \times 12,763,470 - 11,944^2} = \frac{467,624}{10,502,504}$$

$$= 0.044525$$

and: $a = \bar{y} - b\bar{x}$

$$= \frac{462.1}{12} - 0.044525 \times \frac{11,994}{12} = 5.9944$$

Hence the regression line is $y = -5.9944 + 0.044525x$



If the sales are £1,000 million, then x = 1,000 and the regression equation gives:

$$y = -5.9944 + 0.044525 \times 1000 = 38.5306$$

That is, profits = £38.5 million

(b) The regression line can be interpreted as indicating that, within the range of the data, each £1 million of sales generates £44,525 profit.

Assuming there are no changes in background circumstances, the forecast can be considered fairly reliable. First of all, the graph indicates a good correlation between profits and sales and so any forecasts produced by the regression line are likely to be reliable. (In fact the correlation coefficient, r, is approximately 0.8). Further, the forecast is an interpolation (the x-value line within the range of the given data), which is a further indication of reliability.

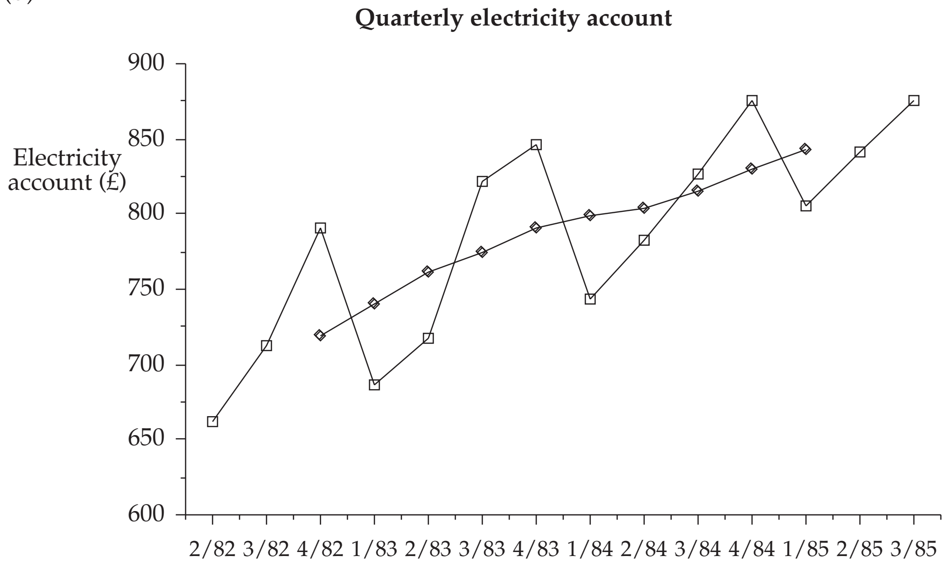
Part 4

Question 1

(a)(c)

		Account (£) (y)	Moving total	Moving average	Centred moving average (t)	Seasonal variation	Deseasonalised data
1982	Q2	662				-39	701
	Q3	712	2850	712.50		45	667
	Q4	790	2906	726.50	719.5	51	739
1983	Q1	686	3015	753.75	740.1	-56	742
	Q2	718	3071	767.75	760.8	-39	757
	Q3	821	3128	782.00	774.9	45	776
	Q4	846	3192	798.00	790.0	51	795
1984	Q1	743	3198	799.50	798.8	-56	799
	Q2	782	3228	807.00	803.3	-39	821
	Q3	827	3290	822.50	814.8	45	782
	Q4	876	3350	837.50	830.0	51	825
1985	Q1	805	3399	849.75	843.6	-56	861
	Q2	842				-39	881
	Q3	876				45	831

(b)



(d) The difference in values between the first and last trend values is:

$843.6–719.5 = 124.1$

Thus, the average increase between trend values is

$\frac{124.1}{9} = 14$ approximately.

Estimated trend values

1985 Q4: $843.6+3(14) = 886$

1986 Q1: $886+14 = 900$.

Therefore, adding the respective seasonal variation value, we have:

Forecast values

1985 Q4: $886 + 51 = £937$

1985 Q4: $900 – 56 = £844$.

Question 2

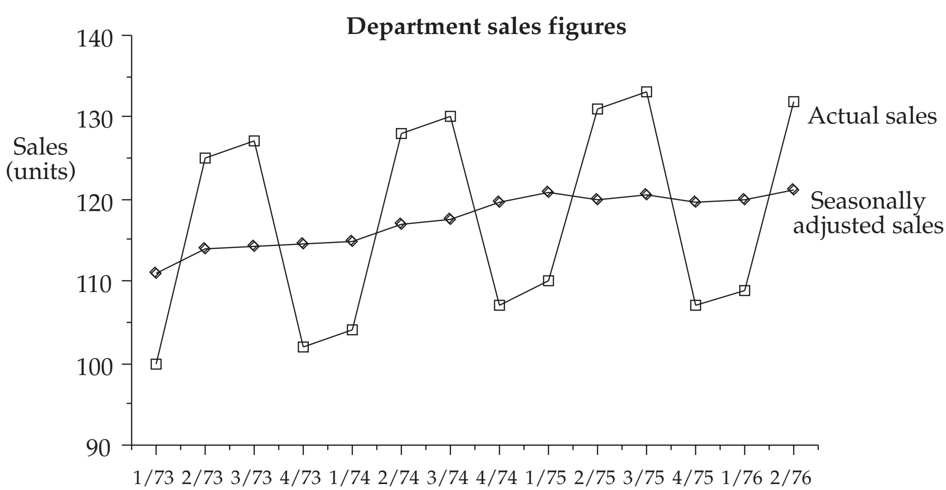
(a) and (b) Main table of calculations:

		Moving total	Moving average	Centred moving average (t)	Deviation (y-t)	Seasonal variation (s)	Deseasonalised data (y-s)
73 1	100					-10.9	110.9
2	125	454	113.50			11.0	114.0
3	127	458	114.50	114.000	13.000	12.6	114.4
4	102	461	115.25	114.875	-12.875	-12.7	114.7
74 1	104	464	116.00	115.625	-11.625	-10.9	114.9
2	128	469	117.25	116.625	11.375	11.0	117.0
3	130	475	118.75	118.000	12.000	12.6	117.4
4	107	478	119.50	119.125	-12.125	-12.7	119.7
75 1	110	481	120.25	119.875	-9.875	-10.9	120.9
2	131	481	120.25	120.250	10.750	11.0	120.0
3	133	480	120.00	120.125	12.875	12.6	120.4
4	107	481	120.25	120.125	-13.125	-12.7	119.7
76 1	109					-10.9	119.9
2	132					11.0	121.0

Seasonal variation calculations

	Q1	Q2	Q3	Q4	
1973			13.000	–12.875	
1974	–11.625	11.375	12.000	–12.125	
1975	–9.875	10.750	12.875	–13.125	
Totals	–21.500	22.125	37.875	–38.125	
Averages (1D)	–10.8	11.1	12.6	–12.7	(Total=+0.2)
Adjustments	–0.1	–0.1	–	–	
Seasonal variation	–10.9	11.0	12.6	–12.7	

(c)



(d) Both trend and seasonally adjusted values show a steady increase up to the beginning of 1975, when they levelled out. Seasonal patterns are well marked and continue throughout the whole period. Adjustments to seasonal averages were very small, leading to the conclusion that there was very little residual variation other than random factors.

Question 3

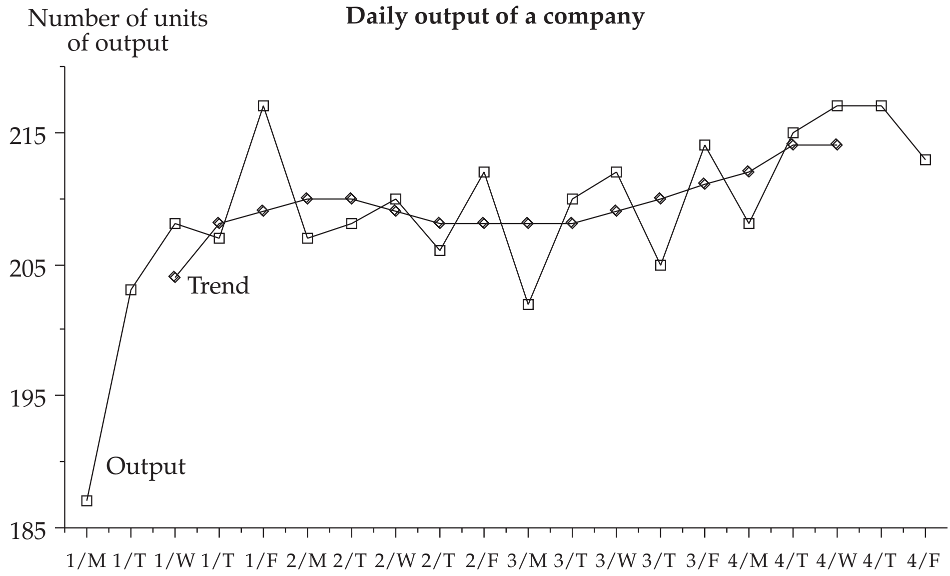
(a) Table of main calculations.

		Output (y)	5-day moving total	Trend (t)	Variation (y-t)
Week 1	Mon	187			
	Tue	203			
	Wed	208	1022	204.4	3.6
	Thu	207	1042	208.4	-1.4
	Fri	217	1047	209.4	7.6
Week 2	Mon	207	1049	209.8	-2.8
	Tue	208	1048	209.6	-1.6
	Wed	210	1043	208.6	1.4
	Thu	206	1038	207.6	-1.6
	Fri	212	1040	208.0	4.0
Week 3	Mon	202	1042	208.4	-6.4
	Tue	210	1041	208.2	1.8
	Wed	212	1043	208.6	3.4
	Thu	205	1049	209.8	-4.8
	Fri	214	1054	210.8	3.2
Week 4	Mon	208	1059	211.8	-3.8
	Tue	215	1071	214.2	0.8
	Wed	217	1070	214.0	3.0
	Thu	217			
	Fri	213			

(b) See graph on opposite page

(c)

	Mon	Tue	Wed	Thu	Fri	
Week 1			3.6	–1.4	7.6	
Week 2	–2.8	–1.6	1.4	–1.6	4.0	
Week 3	–6.4	1.8	3.4	–4.8	3.2	
Week 4	–3.8	0.8	3.0			
Totals	–13.0	1.0	11.4	–7.8	14.8	
Averages	–4.3	0.3	2.9	–2.6	4.9	(Total–1.2)
Adjustments	–0.3	–0.2	–0.2	–0.2	–0.3	
Daily variation	–4.6	0.1	2.7	–2.8	4.6	



(d) Using the calculated (moving average) trend values, the average daily increase in trend can be calculated as: $\dfrac{214.0 - 204.4}{15} = 0.64.$

Trend value for Week 5 (Monday) = 214.0 + 3(0.64) = 215.9 (1D).

Trend value for Week 5 (Tuesday) = 215.9 + 0.64 = 216.6 (1D).

Forecast output for Week 5 (Monday) = 215.9 – 4.6 = 211 (to nearest unit).

Forecast output for Week 5 (Tuesday) = 216.6 + 0.1 = 217 (to nearest unit).

(e) No forecast can ever be confidently made, since it is based only on past evidence and there can be no guarantee that the trend projection is accurate or that the daily variation figures used will be valid for future time points. Only general experience and a particular knowledge of the given time series environment would help further in determining the accuracy of the given forecasts.

Question 4

(a) Trend, seasonal and residual variation. Residual variation contains both random and possible long-term cyclic variations.

(b) (i) Main table of calculations.

	Number of unemployed (y)	Totals of 4	Moving average	Centred moving average (t)	(y-t)
79 Jan	22				
Apr	12	175	43.75		
Jul	110	174	43.50	43.625	66.375
Oct	31	188	47.00	45.250	-14.250
80 Jan	21	228	57.00	52.000	-31.000
Apr	26	267	66.75	61.875	-35.875
Jul	150	296	74.00	70.350	79.625
Oct	70	306	76.50	75.250	-5.250
81 Jan	50	302	75.50	76.000	-26.000
Apr	36	342	85.50	80.500	-44.500
Jul	146				
Oct	110				

Calculations for seasonal variation:

	Jan	Apr	Jul	Oct
1979			66.375	–14.250
1980	–31.000	–35.875	79.625	–5.250
1981	–26.000	–44.500		
Totals	–57.000	–80.375	146.000	–19.500
Averages	–28.2	–40.2	73.0	–9.8 (Tot=–5.5)
Adjustments	+1.4	+1.4	+1.4	=1.3
Seasonal variation	–27.1	74.4	74.4	–8.5

(ii) Seasonally adjusted values: 1981, Jan = $y - s$ = 50 – (–27.1) = 77.1
1981, Apr = 36 – (–38.3) = 74.3

Part 5

Question 1

(i) An index number enables the value of some economic commodity to be compared over some defined time period. It is expressed in percentage terms, using a base of 100.

(ii) (iii) are shown in the following table:

Year	Average salary (£)	Year on year increase (%)	Retail Price Index (1975=100)	Year on year increase (%)	Revalued salary (1985 base)
1977	9500		135.1		19464.10
1978	10850	14.2	146.2	8.2	20542.27
1979	13140	21.1	165.8	13.4	21936.98
1980	14300	8.8	195.6	18.0	20236.40
1981	14930	4.4	218.8	11.9	18887.68
1982	15580	4.4	237.7	8.6	18142.80
1983	16200	4.0	248.6	4.6	18037.65
1984	16800	3.7	261.0	5.0	17817.01
1985	17500	4.2	276.8	6.1	17500.00

Note: $£17{,}817.01 = \dfrac{16800 \times 276.8}{261.0}$; $£18{,}037.65 = \dfrac{16200 \times 276.8}{248.6}$; etc

(iv) Except for the first two years, the increase in prices has outstripped the increase in salary. The revalued salary shows that (in real terms) the systems analysts are being paid less now than in any of the previous nine years.

Question 2

(a) The Retail Prices Index can be used by retailers to compare their own average price increases with those that consumers are subject to. Wage-earners often use the RPI (although the Tax and Price Index is more relevant) to compare their wage increases with the increases in prices. Trades Union use the value of the RPI to negotiate price increases with employers.

The Producer Price Indices can be used by retailers to compare the prices they are paying for their goods. It can also be used by consumers as a long term warning (nine months or so) of trends that will inevitably be felt in the RPI.

The Index of Output of the Production Industries is used as a general guide to measure the changes in the level of production in the UK.

(b) Putting July 1979 as year 0 etc, we have:

	$W_0 \times E_0$	$W_1 \times E_0$	$W_0 \times E_2$	$W_2 \times E_2$
Q	300	350	240	320
R	120	130	180	210
S	140	170	70	90
T	90	110	180	240
Total	650	760	670	860

(i) Laspeyre index for year 1 = $L_1 = \dfrac{\sum W_1 E_0}{\sum W_0 E_0} = \dfrac{760}{650} \times 100 = 116.9$

(ii) Paasche index for year 2 = $P_2 = \dfrac{\sum W_2 E_2}{\sum W_0 E_2} = \times 100 = 128.4$

(iii) The indices given can be base-changed to 1979. Thus:

$$L_{1/0} = \frac{187.4}{156.3} \times 100 = 119.9; \quad L_{2/0} = \frac{203.4}{156.3} \times 100 = 130.1$$

These indices, when compared with the company's indices, show that the wage rates of the company are lagging slightly behind the Chemical and Allied Industry's rates by about two points.

Question 3

(a) In 1974 (on average, per week) 4 hours overtime was worked, which is equivalent to 4 ×1.5=6 normal hours. Thus, dividing the average weekly earnings by 46 (the equivalent normal hours worked per week) will give the normal rate per hour as 40.19÷46=£0.87. Multiplying this by 40 will thus yield the normal weekly rate of 40× 0.87=£34.95. This must be done for each year.

i.e. average normal weekly hours = 40 + (ave hours worked – 40) × 1.5

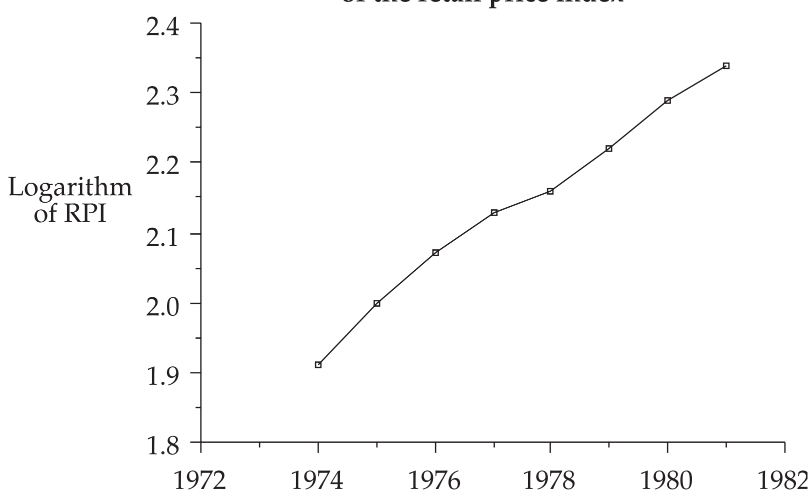
and normal weekly rate = ave earnings × $\dfrac{40}{\text{average normal weekly hours}}$

Year	Average weekly earnings	Average hours worked	Equivalent normal weekly hours	Normal weekly rate
1974	40.19	44	46	34.95
1975	52.65	45	47.5	44.34
1976	62.03	45	47.5	52.24
1977	70.20	46	49	57.31
1978	76.83	46	49	62.72
1979	91.90	46	49	75.02
1980	107.51	45	47.5	90.53
1981	121.95	43	44.5	109.62

(b)

RPI	80.5	100.0	116.5	135.0	146.2	165.8	195.6	218.9
Log	1.91	2.00	2.07	2.13	2.16	2.22	2.29	2.34

Semi-logarithmic graph of the retail price index



The above logarithms are plotted against the relevant year to form a semi-logarithmic graph which is shown in the figure above.

(c) Since the semi-graph in Figure 1 is an approximate straight line, this demonstrates that the rate of increase of the RPI is constant.

(d) A deflated normal weekly rate can be obtained by dividing each normal weekly rate by the value of the RPI for that year and multiplying back by 100 to bring the value back to the correct form.

e.g. deflated normal weekly rate for 1974 = $34.95 \times \dfrac{100}{80.5} = £43.42$

Year	1974	1975	1976	1977	1978	1979	1980	1981
Deflated normal weekly rate (£)	43.42	44.34	44.84	42.45	42.90	45.25	46.28	50.08

(e) If an index of real wages is calculated, it will enable a comparison between the increase in prices and real wages to be made.

Question 4

Item (w)	Weight (I)	Index (wI)	(1) (wI)	(2) (wI)	(3)
Mining and quarrying	41	361	14801		
Manufacturing					
Food, drink and tobacco	77	106	8162	8162	8162
Chemicals	66	109	7194	7194	7194
Metal	47	72	3384	3384	3384
Engineering	298	86	25628	25628	25628
Textiles	67	70	4690	4690	4690
Other manufacturing	142	91	12922	12922	12922
Construction	182	84	15288	15288	
Gas, electricity and water	80	115	9200	9200	
	1000		101269	86468	61980

(i) (1) All industries index is given by: $\dfrac{101269}{1000} = 101.3$

(2) All industries except mining and quarrying index is: $\dfrac{86468}{1000-41} = 90.2$

(3) Manufacturing industries index is: $\dfrac{61980}{1000-41-182-80} = 88.9$

(ii) The high mining and quarrying index of 361 was severely offset by its small weight in the relatively low value of 101.3 for the overall index in (1). However, the index of only 90.2 in (2) shows the significance of mining and quarrying (particularly North Sea oil) to industrial production in the UK. The low manufacturing index of 88.9 in (3) is due to the fact that the three largest weights are assigned to relatively low indices.

Question 5

FOOD: The movement in these weights can be accounted for by the increased affluence of our society, which results in a much greater pool of disposable income left after the basic necessities (of which food is one of the most important) have been acquired. Also, since it is reasonable to assume that we are not buying less food in

1981 than we were in 1961, it means that (in relative terms) food is now cheaper. Since food has such a high weighting, the value of its index will bear the most significant effect on the RPI itself.

HOUSING: The increase in expenditure is probably due to two factors. First, a significant part of the extra disposable income is being spent on housing. Second, housing is more expensive in real terms. Changes in such things as mortgage rates and rents will now have a more significant effect on the RPI than was previously the case.

CLOTHING: The decrease in expenditure is probably due to cheap imports, since again we can only suppose that we are buying at least as much clothing in 1981 as we were in 1961.

TRANSPORT: The dramatic increase in transport costs are probably due to the increased mobility we now have as a society. We travel much further both to and from our place of occupation and also for leisure and recreation purposes. Changes in petrol prices and car tax will now have much more effect on the RPI than they did previously.

Part 6

Question 1

(a) Set P = £40,000, for convenience.

Time 0: amount owed = P.

After 1 quarter, 4% is added to amount owing, giving $P(1 + 0.04) = PR$

X is paid, leaving amount owed = $PR - X$.

After 2 quarters 4% is added to amount owing, and X is paid.

Amount now owed $= (PR - X)R - X$

$= PR^2 - XR - X$ or £$(40{,}000\,R^2 - XR - X)$

(b) After 3 quarters, amount owed $= (PR^2 - XR - X) - R - X$

$= PR^3 - XR^2 - XR - X$

And so on, until, after 80 quarters, the amount owed

$= PR^{80} - XR^{79} - XR^{78} - \ldots -X$

$= PR^{80} - X(R^{79} + R^{78} + \ldots + R + 1)$

$= PR^{80} - X\left(\dfrac{R^{80}-1}{R-1}\right)$ from the geometric progression formula.

Now, as the mortgage is to be paid off in this period, this amount owed must be zero, and so:

$$PR^{80} = X\left(\frac{R^{80}-1}{R-1}\right)$$

and: $$P.R^{80}.\left(\frac{R-1}{R^{80}-1}\right) = X$$

$$0.0418P = X \text{ (from tables)} \qquad [*]$$

Since P = £40,000, the quarterly repayment is £1,672.56.

(c) Using [*], if P is doubled from £40,000 to £80,000, the factor 0.0418 would be unaltered, and so the repayment figure would double to $2X$.

Question 2

(a) We are given that: P=12000; i=0.15; n=5. Putting the amortization payment as A, we must have that:

$$12000 = \frac{A}{1.15} + \frac{A}{1.15^2} + ... + \frac{A}{1.15^5}$$
$$= A(0.86957 + 0.75614 + 0.65752 + 0.57175 + 0.49718)$$
$$= A(3.35216)$$

Therefore, $A = \frac{12000}{3.35216} = 3579.79$. That is, amortization payment = £3579.79

The amortization schedule is tabulated as follows:

Year	Amount outstanding (beginning)	Interest	Payment
1983	12000.00	1800.00	3579.79
1984	10220.21	1533.03	3579.79
1985	8173.45	1226.02	3579.79
1986	5819.68	872.95	3579.79
1987	3112.84	466.93	3579.79
1988	(0.02)		

(b) Here, there are two interest rates. The investment rate, j=0.1 and the borrowing rate, i=0.15. Also, P=12000 and n=5.

The calculations for the sinking fund payment (ordinary annuity) is given in the following.

The debt will amount to 12000(1.15)5 = £24,136.29 after 5 years. Thus, the sinking fund must mature to this amount. If A is the annual deposit into the fund, then we must have that:

$$24136.29 = A + A(1.1) + A(1.1)\ 2 + A(1.1)\ 3 + A(1.1)\ 4$$
$$= A(1 + 1.1 + 1.21 + 1.331 + 1.4641) = A(6.1051)$$

Therefore, $A = \frac{24136.29}{6.1051} = £3953.46$

The Sinking Fund schedule is tabulated as follows:

Year	Debt outstanding	Interest on debt	Deposit	Amount in fund	Interest on fund
1983	12000.00	1800.00	0	0	0
1984	13800.00	2070.00	3953.46	3953.46	395.35
1985	15870.00	2380.50	3953.46	8302.27	830.23
1986	18250.50	2737.58	3953.46	13085.96	1308.60
1987	20988.08	3148.21	3953.46	18348.02	1834.80
1988	24136.29		3953.46	24136.28	

(c) Discounted cash flow table for calculation of NPV:

Year	Net cash flow	Discount Factor (10%)	Discount Factor (15%)	Present value (10%)	Present value (15%)
1983	(12000)	1.0000	1.0000	(12000.00)	(12000.00)
1984	6600	0.9091	0.8696	6000.06	5739.36
1985	6000	0.8264	0.7561	4958.40	4536.60
1986	4500	0.7513	0.6575	3380.85	2958.75
1987	(1000)	0.6830	0.5718	(683.00)	(571.80)
1988	(2600)	0.6209	0.4972	(1614.34)	(1292.72)
			NVP	41.97	(629.81)

(d) Using the formula method to determine the IRR, we have:

$I_1=10$; $N_1=41.97$; $I_2=15$; $N_2=-629.81$

and $\text{IRR} = \dfrac{N_1I_2 - N_2I_1}{N_1 - N_2} = \dfrac{(41.97)(15) - (-629.81)(10)}{41.97 - (-629.81)} = \dfrac{629.55 + 6298.1}{671.78}$

giving IRR = 10.3%.

The IRR gives the rate which makes NPV=0.

Question 3

(a) This can be calculated using a schedule as follows:

Year	Amount in fund	Interest	Total in fund	
1	10000	1000	11000	
2	21000	2100	23100	
3	33100	3310	36410	
4	46410	4641	51051	= value

(b) To calculate present value:

Year	Net savings	11% discount factor	Present value
1	2000	0.9009	1801.81
2	2000	0.8116	1623.20
3	2000	0.7312	1462.40
4	2000	0.6587	1317.40
			6204.80

(c) This can again be calculated using a schedule:

Year	Total amount invested	Interest (10%)	Total	
1	1000.00	100.00	1100.00	
2	1600.00	160.00	1760.00	
3	2260.00	226.00	2486.00	
4	2986.00	298.60	3284.60	
5	3784.60	378.46	4163.06	= acc sum

(d) This requires the value of an amortization annuity. Given that: $P = 20000$; $n = 20$; $i = 0.14$. If the yearly repayment is A, then:

$$20000 = \frac{A}{1.14} + \frac{A}{1.14^2} + \ldots + \frac{A}{1.14^{20}}$$
$$= A.\left(\frac{1}{1.14} + \frac{1}{1.14^2} + \ldots + \frac{1}{1.14^{20}}\right)$$

But the terms in the bracket form a gp with a = $\frac{1}{1.14}$ and $r = \frac{1}{1.14}$.

The sum to 20 terms of this gp is given by:

$$\frac{\frac{1}{1.14}\left[1-\left(\frac{1}{1.14}\right)^{20}\right]}{1-\frac{1}{1.14}} = \frac{0.8772 \times 0.9272}{0.1228} = 6.6233$$

Therefore, $20000 = A(6.6233)$, giving $A = \frac{20000}{6.6233} = £3019.65$

Question 4

(a) The amount of the mortgage must sum to the present value of all the payments made. If A is the annual payment, then:

$$10000 = \frac{A}{1.12} + \frac{A}{1.12^2} + \ldots + \frac{A}{1.12^5}$$

Thus: $10{,}000 = A(0.8929 + 0.7972 + 0.7118 + 0.6355 + 0.5674)$
$= 3.6048A$

Therefore, $A = \frac{10000}{3.6048} = £2774.10$

(b)

Year	Outstanding debt	Interest paid	Payment	Principal repaid
1	£10,000.00	£1,200.00	£2,774.10	£1,574.10
2	£8,425.90	£1,011.11	£2,774.10	£1,762.99
3	£6,662.91	£799.55	£2,774.10	£1,974.55
4	£4,688.37	£562.60	£2,774.10	£2,211.49
5	£2,476.87	£297.22	£2,774.10	£2,476.87

(c) After 5 years, the debt will amount to: $£10{,}000(1.12)^5 = £17{,}623.42$, which is therefore the amount that the fund must mature to. Putting A as the annual premium into the fund, gives:

$$17{,}623.42 = A(1.15) + A(1.15)^2 + \ldots + A(1.15)^5$$
$$= A(1.15 + 1.3225 + 1.5209 + 1.7490 + 2.0114)$$
$$= 7.7538A$$

Thus, $A = £2272.89$

(d)

Year	Debt Outstanding	Interest on Debt	Deposit	Amount in Fund	Interest on Fund
1	£10,000.00	£1,200.00	£2,272.89	£2,272.89	£340.93
2	£11,200.00	£1,344.00	£2,272.89	£4,886.72	£733.01
3	£12,544.00	£1,505.28	£2,272.89	£7,892.62	£1,183.89
4	£14,049.28	£1,685.91	£2,272.89	£11,349.41	£1,702.41
5	£15,735.19	£1,888.22	£2,272.89	£15,324.71	£2,298.71
6	£17,623.42			£17,623.42	

Question 5

(a) This question refers to repayments including both capital and interest; that is, amortization. With $P = 100{,}000$ $n = 4$ and $i = 0.12$, the amortization payment, A say, must satisfy:

$$100{,}000 = \frac{A}{1.12} + \frac{A}{1.12^2} + \frac{A}{1.12^3} + \frac{A}{1.12^4}$$

$$= A(0.89286 + 0.79719 + 0.71178 + 0.63552)$$

$$= A(3.03735).$$

Therefore, $A = \dfrac{100000}{3.03735} = £32{,}923.44.$

(b) The reducing balance depreciation formula can be used, namely: $D = B.(1- i)^n$ where $D = 1000$, $B = 50000$ and $n = 5$. Here, i is to be determined.

Re-arranging gives $\dfrac{D}{B} = (1- i)^n$ or $1- i = \left(\dfrac{D}{B}\right)^{\frac{1}{n}}$

So that: $1- i = \left(\dfrac{1000}{50000}\right)^{\frac{1}{5}} = (0.02)^{0.2} = 0.457$

Therefore $i = 1–0.457 = 0.543 = 54.3\%$

(c) Working in time periods of one quarter (year), we have $A = P.(1+i)^n$ where: $P = 1000$, $A = 3000$

and $i = \dfrac{0.12}{4} = 0.03$ (per quarter).

Re-arranging gives: $(1+ i)^n = \dfrac{A}{P}$

and substituting: $(1.03)^n = \dfrac{3000}{1000} = 3$

Using logarithms: $n.\log(1.03) = \log(3)$

Therefore $n = \dfrac{\log(3)}{\log(1.03)} = \dfrac{0.4771}{0.01284} = 37$ (approximately).

Thus, number of years $= \dfrac{37}{4} = 9.25.$

Question 6

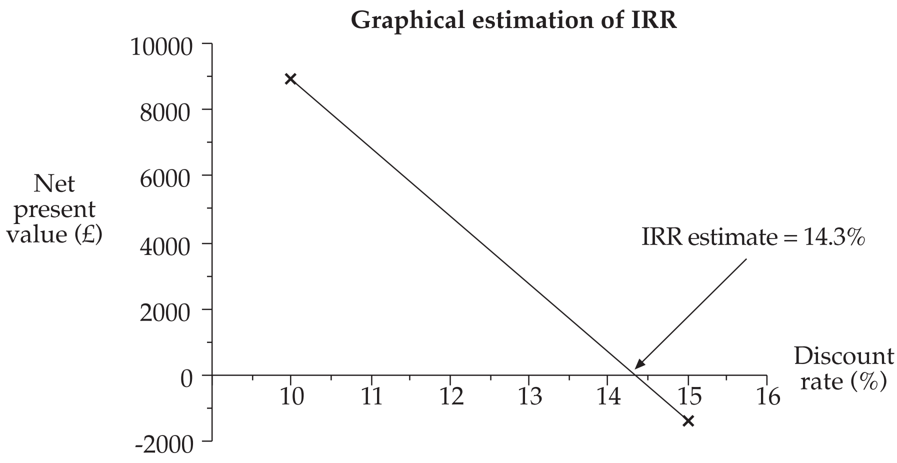
(a) Cash flow table:

End year	Machinery	Maintenance	Revenue	Net
0	(75000)			(75000.00)
1		(1000)	20000.00	19000.00
2		(1100)	21500.00	20400.00
3		(1210)	23112.50	21902.50
4		(1331)	24845.94	23514.94
5			26709.38	26709.38
6	1250			1250.00

(b) and (c) Discounted cash flow table:

Year	Net flow	Discount factor (10%)	Discount factor (15%)	Present value (10%)	Present value (15%)
0	(75000.00)	1.0000	1.0000	(75000.00)	(75000.00)
1	19000.00	0.9091	0.8696	17272.90	16522.40
2	20400.00	0.8264	0.7561	16858.55	15424.44
3	21902.50	0.7513	0.6575	16455.34	14400.89
4	23514.94	0.6830	0.5718	16060.70	13445.84
5	26709.38	0.6209	0.4972	16583.85	13279.90
6	1250.00	0.5645	0.4323	705.62	540.37
				8936.99	(1386.15)

(d)



(e) The value of 14.3% for the IRR can be interpreted as the rate of return that the project earns.

Question 7

The calculations for the net present value for each one of the three decisions are tabulated below.

	Decision (i)		Decision (ii)		Decision (iii)	
Discount factor at 12%	Net cost	Present value	Net cost	Present value	Net cost	Present value
1	95000	95000	52000	52000	32000	32000
0.8929	20000	17857	27000	24107	32000	28571
0.7972	20000	15944	27000	21524	32000	25510
0.7118	20000	14236	27000	19218	32000	22777
0.6355	20000	12710	27000	17159	32000	20337
0.5674	–10000	–5674	–10000	–5674	0	0
		150073		128334		129195

Decision (ii) is the least costly and thus, on pure financial grounds, should be chosen. However, this does not take into account the usefulness of the computer over the next five years. For example, could the old machine run newly developed software?

Question 8

(a) (i) After four years (i.e. 8 × 6 months):
$A = P.(1+i)^n = 2750(1.035)^8 = £3621.22.$

(ii) $(1.035)^2 - 1 = 1.071 - 1 = 0.071$ or 7.1%.

(b) (i) The amortization method of debt repayment is required here. We are given: P=37500, i=0.12 and n=5. So, if A is the annual payment to be found, then:

$$37500 = \frac{A}{1.12} + \frac{A}{1.12^2} + \frac{A}{1.12^3} + \frac{A}{1.12^4} + \frac{A}{1.12^5}$$
$$= A(0.89286 + 0.79719 + 0.71178 + 0.63552 + 0.56743)$$
$$= A(3.60478)$$

Thus, $A = \dfrac{37500}{3.60478} = £10{,}402.86.$

(ii) Amortization schedule:

Year	Amount outstanding	Interest paid	Payment
1	37500.00	4500.00	10402.86
2	31597.14	3791.66	10402.86
3	24985.94	2998.31	10402.86
4	17581.39	2109.77	10402.86
5	9288.29	1114.59	10402.86
Balance	0.03		

(c) (i) Borrowing rate, i = 12% = 0.12; Investment rate, j = 8% = 0.08. Principal amount borrowed, P = £37500, n = 5 (years).

Notice that the payments into the fund are in advance. i.e. the payments form a due annuity.

The debt will amount to £37500(1.12)5 = £66,087.81 after 5 years. Thus the fund must mature to this amount. Putting A as the annual deposit into the fund, we must have that:

$$\begin{aligned}66087.81 &= A(1.08) + A(1.08)^2 + A(1.08)^3 + A(1.08)^4 + A(1.08)^5\\ &= A(1.08 + 1.1664 + 1.25971 + 1.36049 + 1.46933)\\ &= A(6.33593).\end{aligned}$$

Therefore, $A = \dfrac{66087.81}{6.33593} = £10{,}430.64$.

(ii) Schedule:

Year	Debt outstanding	Interest paid	Deposit	Amount in fund	Interest earned
1	37500.00	4500.00	10430.64	10430.64	834.45
2	42000.00	5040.00	10430.64	21695.73	1735.66
3	47040.00	5644.80	10430.64	33862.03	2708.96
4	52684.80	6322.18	10430.64	47001.63	3760.13
5	59006.98	7080.84	10430.64	61192.40	4895.39
6	66087.82			66087.79	

Question 9

(a)

Time (year)	Receipt (£)	Present value at start of annuity (£)
0.5	1,500	$\dfrac{1500}{1045}$
1.0	1,500	$\dfrac{1500}{1045^2}$
1.5	1,500	$\dfrac{1500}{1045^3}$
10.0	1,500	$\dfrac{1500}{1045^{20}}$

The net present value of the annuity is thus a geometric progression with $n = 20$ terms, first term 'a' $= \dfrac{1500}{1.045}$ and ratio 'r' $= \dfrac{1}{1.045}$

The sum is therefore $= \dfrac{\dfrac{1500}{1.045}\left(1-\dfrac{1}{1.045^{20}}\right)}{1-\dfrac{1}{1.045}}$

It is therefore worth paying up to £19,512, for the annuity.

(b) If we denote the quarterly amount by £X

after 1 quarter, value of amount paid $= X$

after 2 quarters, value of amounts paid $= X(1 + 0.025) + X$

after 3 quarters, value of amounts paid $= X(1 + 0.025)^2 + X$

after 100 quarters:

$$X(1.025)^{99} + X(1.025)^{98} + \ldots + X = \frac{X\left(1.025^{100} - 1\right)}{1.025 - 1} = 432.5488X$$

This must balance the value the original loan has reached after 100 quarters at 2.5% per quarter.

By the compound interest formula, this is 50,000 $(1 + 0.025)^{100}$ = £590,686

Hence: $432.5488X = 590{,}686$ giving $X = 1365.5939$

Thus the quarterly repayments are £1,365.59

(c) As in (b), after 59 months, the value of the scheme will have reached

$300\,(1.01)^{59} + 300\,(1.01)^{58} + \ldots + 300$

(Note that £300 is paid immediately, at time 0, in this case)

This value $= \dfrac{300\left(1.01^{60} - 1\right)}{1.01 - 1} = 24{,}500.91$

Now, adding on the interest for the final month, the final value is

$24{,}500.91 \times 1.01$ = £24,746 (to nearest £)

The real value of the investment is $\dfrac{24{,}746}{1.05^5}$ = £19,389 (to nearest £)

(d) In (a), the administrative and other charges usually involved with annuities may vary in the future.

In (b), it is very common for mortgage interest rates to vary, thereby varying the quarterly instalments.

In (c), the scheme may pay more than the minimum 1% or the assumption of 5% inflation may prove accurate.

Question 10

(a) The NPVs at 10% cost of capital for the two machines are shown

Machine A		Net Flow (£000)	10% Discount factor	Present value (£000)
Year	Note			
0		-100	1.0000	-100.00
1	a	-60	0.9091	-54.55
2		40	0.8264	33.06
3		40	0.7513	30.05
4		40	0.6830	27.32
5		40	0.6209	24.84
6	b	60	0.5645	33.87
		Net present value		-5.41

Machine B		Net Flow (£000)	10% Discount factor	Present value (£000)
Year	Note			
0		-120	1.0000	-120.00
1	a	-70	0.9091	-63.64
2		50	0.8264	41.32
3		50	0.7513	37.57
4		50	0.6830	34.15
5		50	0.6209	31.05
6	b	74	0.5645	41.77
		Net present value		2.22

Note a: Includes balance of cost of machine. Note b: Includes scrap value.

(b) From the figures shown, it is clear that machine B should be chosen since it has the higher NPV. It should be noted that machine A does not even return as much as 10% on the overall investment.

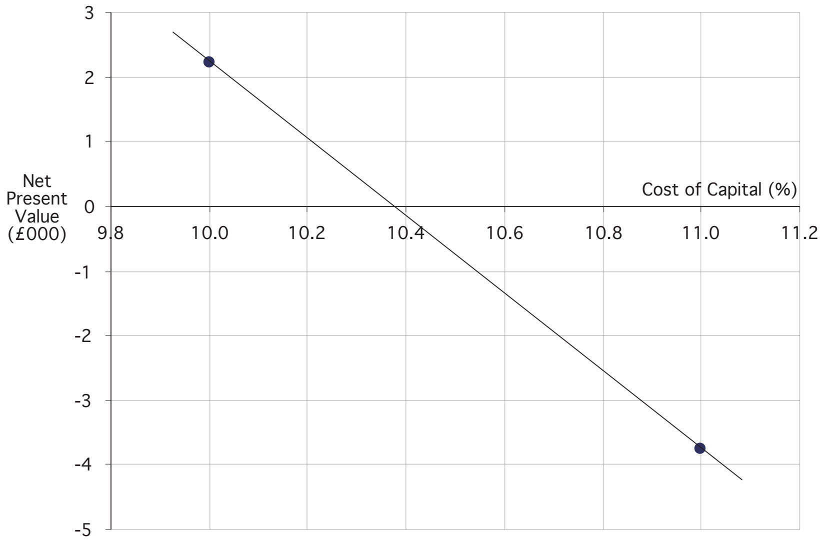
The assumptions made in the above recommendation are that the company can can afford the outlays inherent in the above structure. In the comparison above, no account has been taken of relative risks and it is clear that machine B, being the more expensive, is the riskier investment.

Machine B		Net Flow	11% Discount	Present value
Year	Note	(£000)	factor	(£000)
0		-120	1.0000	-120.00
1	a	-70	0.9009	-63.06
2		50	0.8116	40.58
3		50	0.7312	36.56
4		50	0.6587	32.94
5		50	0.5935	29.67
6	b	74	0.5346	39.56
		Net present value		-3.75

Note a: Includes balance of cost of machine. Note b: Includes scrap value.

(c) The table above shows the net flows at an 11% cost of capital.

The diagram below shows the NPVs obtained from 10 and 11% costs of capital plotted and the estimate of the IRR is seen to be approximately 10.4%.



Part 7

Question 1

a), b) Materials cost £0.50 per poster, that is $0.50N$ (£); Labour costs £15 per hour, and N posters take $\dfrac{N}{300} + 2$ hours to produce. Thus the labour costs are:

$$15.\left(\frac{N}{300}+2\right) = \frac{N}{20} + 30 \text{ (£)}$$

Administration costs are £10 per hundred posters plus £50; that is:

$$10.\frac{N}{100} + 50 = \frac{N}{10} + 50 \text{ (£)}$$

Adding (i), (ii) and (iii), total costs are given by:

$$C = 0.50N + \frac{N}{20} + 30 + \frac{N}{10} + 50 = 0.65N + 80 \text{ (£)}$$

Hence producing 1,000 posters will cost: $0.65 \times 1000 + 80 = £730$

The formula for C indicates a fixed cost of £80 and a variable cost of £0.65 per poster produced.

(c) i. If the cost is £500, then $500 = 0.65N + 80$. Hence $= \dfrac{420}{0.65} = 646$ posters

ii. If the cost is N, then $N = 0.65\text{N} = 80$.

Hence $N = \dfrac{80}{0.35} = 229$ posters

Question 2

(a) We need to solve the equation $P = -32r^2 + 884r - 5985 = 0$

i.e. to solve $32r^2 - 884r + 5985 = 0$

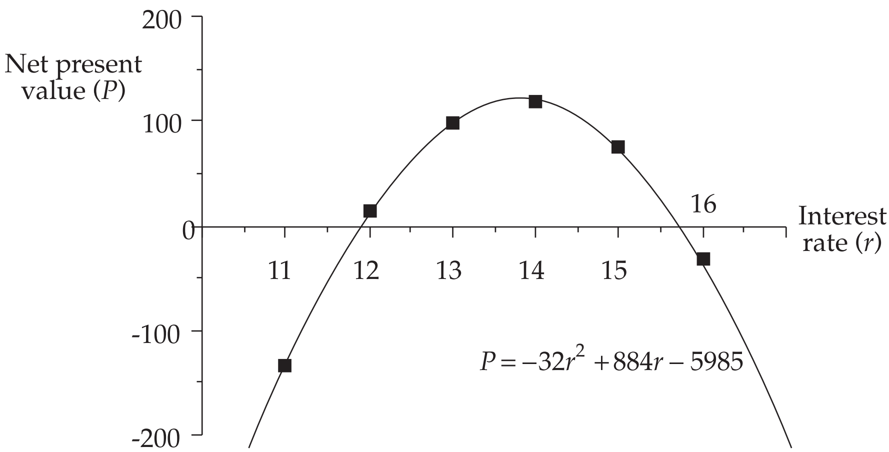
Using the formula, with a=32, b=–884 and c=5985, gives:

$$r = \frac{884 \pm \sqrt{884^2 - 4(32)(5985)}}{2(32)} = \frac{884 \pm 124}{64}$$

That is, $r = 15.75$ or $r = 11.875$

(b) We need to tabulate and plot P for integral values of r between 11 and 16.

$-32r^2$	$884r$	-5985	P
–3872	9724	–5985	–133
–4608	10608	–5985	15
–5408	11492	–5985	99
–6272	12376	–5985	119
–7200	13260	–5985	75
–8192	14144	–5985	–33



(c) With interest rates between 11.875% and 15.75%, the project yields a positive NPV and thus is worthwhile.

(d) Now, $P = -32r^2 + 884r - 5985$.

Thus: $\dfrac{dP}{dr} = -64r + 884$ and $\dfrac{dP}{dr} = 0$ when $64r = 884$. i.e. $r = 13.81$

Also, $\dfrac{d^2P}{dr^2} = -64$, and since this is negative, $r = 13.81$ signifies a max value of P.

Thus, the maximum value of r is $-32(13.81)^2 + 884(13.81) - 5985 = £120.125$.

Question 3

(a) Let n be the old number of passengers and f be the old fare.
Therefore, old revenue $= nf$.
A 30% increase in passengers gives new number of passengers as $1.3\times n$; a 10% decrease in fare gives the new fare as $0.9\times f$.

$$\begin{aligned}\text{Thus, new revenue} &= \text{new number of passengers} \times \text{new fare}\\ &= (1.3)(0.9)nf\\ &= (1.17)nf \ \text{(i.e. 0.17 increase in old revenue).}\end{aligned}$$

Thus percentage increase in revenue is 17%

(b) New fare $= f\left(1-\dfrac{x}{100}\right)$ and new number of passengers $= n\left(1+\dfrac{2x}{100}\right)$

$$\text{Therefore, new revenue} = nf\left(1-\frac{x}{100}\right)\left(1+\frac{2x}{100}\right)$$

$$\begin{aligned}\text{Thus, the multiplier of } nf &= \left(1-\frac{x}{100}\right)\left(1+\frac{2x}{100}\right)\\ &= 1-\frac{x}{100}+\frac{2x}{100}-\frac{2x^2}{10000}\\ &= 1+\frac{x}{100}-\frac{2x^2}{10000}\\ &= 1 + 0.01x - 0.0002x^2.\end{aligned}$$

(c) We need to find the value of x that maximises the multiplier:
$M = 1 + 0.01x - 0.0002x^2$.

But $\dfrac{dM}{dx} = 0.01 - 0.0004x$ and when $\dfrac{dM}{dx} = 0$, $0.0004x = 0.01$.

Thus, the value of x that maximises M is $x = \dfrac{0.01}{0.0004} = 25$.

$$\begin{aligned}\text{When } x=25,\ M &= 1 + (0.01)(25) - (0.0002)(25)^2\\ &= 1 + 0.25 - 0.125\\ &= 1.125\end{aligned}$$

Therefore, percentage increase in revenue is 12.5%.

Question 4

(a) The selling price is £15/unit. Therefore the revenue, $R = 15x$.
The costs, $C = 800 + 5x + 0.009x^2$.
Thus, profit, $P = R - C = 15x - (800 + 5x + 0.009x^2)$
i.e. $P = 10x - 800 - 0.009x^2$.

(i) We require the range of x such that $P \geq 200$.
i.e. such that $10x - 800 - 0.009x^2 \geq 200$ or $10x - 1000 - 0.009x^2 \geq 0$.
Solving $10x - 1000 - 0.009x^2 = 0$ will give the 'critical' points for x.

Here, to use the formula, a=–0.009, b=10 and c=–1000, and:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(-0.009)(-1000)}}{2(-0.009)} = \frac{-10 \pm \sqrt{64}}{-0.018}$$

Therefore x=1000 or x=111.1 (1D).

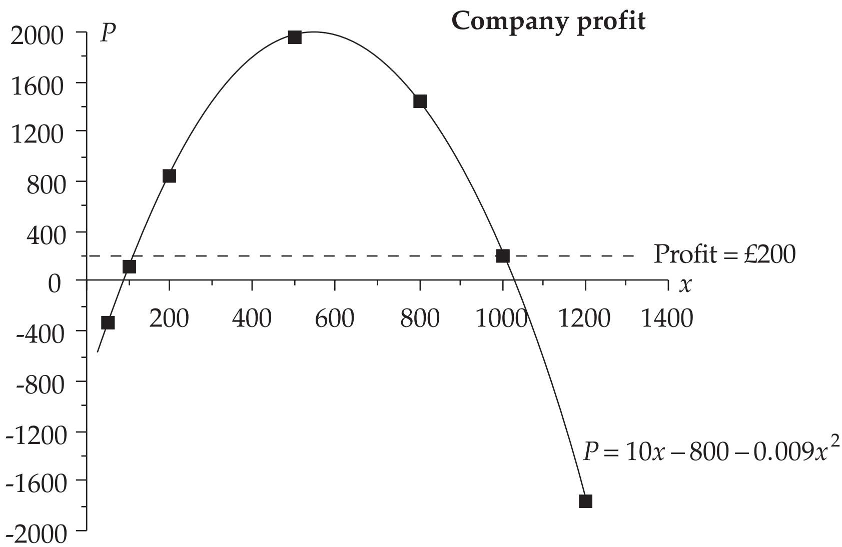
Now, since the graph of $y = 10x - 1000 - 0.009x^2$ is a reverse 'U' (mountain) curve, values of x between 111.1 and 1000 will give $y \geq 0$ as required.

So that, a weekly profit of at least £200 will be provided if the weekly production is between 111.1 and 1000 units.

(ii) The calculations for the graph of $P = 10x - 800 - 0.009x^2$ are tabulated:

x	50	100	200	500	800	1000	1200
$10x$	500	1000	2000	5000	8000	10000	12000
–800	–800	–800	–800	–800	–800	–800	–800
$-0.009x^2$	–22.5	–90	–360	–2250	–5760	–9000	–12960
y	–322.5	110	840	1950	1440	200	–1760

The graph is plotted below.



(b) Using the formula for reducing balance depreciation: $D = B(1-i)$

we have: $23500 = 32000(1-i)^6$ giving $1-i = \left(\frac{23500}{32000}\right)^{\frac{1}{6}}$

Hence, $1-i = 0.9498$. Therefore, i=0.0502. The rate of depreciation is thus 5.02%.

Question 5

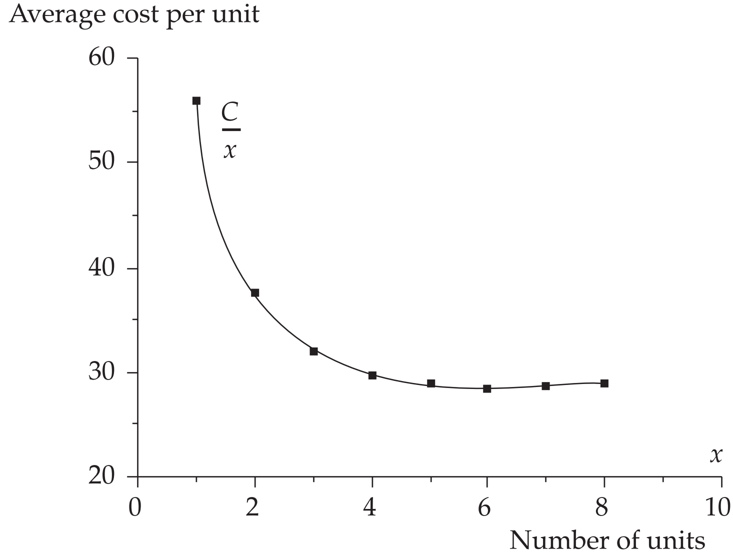
(a) Total cost, $C = x^2 + 16x + 39$.

Average cost per unit = $\frac{C}{x} = x + 16 + \frac{39}{x}$

The calculations for the plot of the average cost per unit are tabulated below.

x	0	1	2	3	4	5	6	7	8
$x + 16$	16	17	18	19	20	21	22	23	24
$\frac{39}{x}$	∞	39	19.5	13	9.75	7.8	6.5	5.6	4.9
$\frac{C}{x}$	∞	56	37.5	32	29.75	28.8	28.5	28.6	28.9

and the graph is shown in the figure.



(b) (i) Demand function: $p = x^2 - 24x + 117$.

Total revenue: $R = p.x = x^3 - 24x^2 + 117x$.

(ii) R is maximised where $\frac{dR}{dr} = 0$ and $\frac{dR}{dr} = 3x^2 - 48x + 117$.

Therefore $\frac{dR}{dr} = 0$ where $3x^2 - 48x + 117 = 0$.

Solving the previous quadratic equation using the formula with a=3, b=–48 and c=117 gives:

$$x = \frac{48 \pm \sqrt{48^2 - 4(3)(117)}}{2(3)} = \frac{48 \pm \sqrt{900}}{6} = \frac{48 \pm 30}{6} \text{ i.e. } x=3 \text{ or } x=13.$$

Now, $\frac{d^2R}{dx^2} = 6x - 48$, and when x=3, $\frac{d^2R}{dx^2} = 6(3) - 48 = -30$.

Thus R is a maximum when x=3.

Price at x=3 is $p(3) = 3^2 - 24(3) + 117 =$ £54/unit.

(iii) $\frac{dp}{dx} = 2x - 24$.

Therefore, elasticity of demand

$$= \left(\frac{x^2 - 24x + 117}{x}\right)\left(\frac{1}{2x - 24}\right) = \frac{x^2 - 24x + 117}{2x(x - 12)}$$

and at x=3 (the maximum revenue point): elasticity

$$= \frac{3^2 - 24(3) + 117}{2(3)(3 - 12)} = \frac{54}{-54} = -1$$

Question 6

(a) The total cost function is obtained by integrating the marginal cost function.

i.e. $C = \int (92 - 2x)dx = 92x - x^2 + K$(where K is the fixed cost).

Fixed cost (K) is given as 800 (£000). Thus $C = 92x - x^2 + 800$.

(b) The total revenue function is obtained by integrating the marginal revenue function.

i.e. $R = \int (112 - 2x)dx = 112x - x^2$ (there is no constant term for revenue since R=0 when x=0)

(c) Now, profit, $P = R - C = 112x - x^2 - (92x - x^2 + 800)$. i.e. $P = 20x - 800$.

The break-even situation is where P=0. i.e. where $20x - 800 = 0$ or x=40.

(d) For maximum revenue, we need $\frac{dR}{dx} = 0$. i.e. $112 - 2x = 0$ or x=56.

Therefore, $R_{max} = R(\text{at } x=56) = 112(56) - 56^2 = 3136$ (£000).

For maximum costs, we need $\frac{dC}{dx} = 0$. i.e. $92 - 2x = 0$ or x=46.

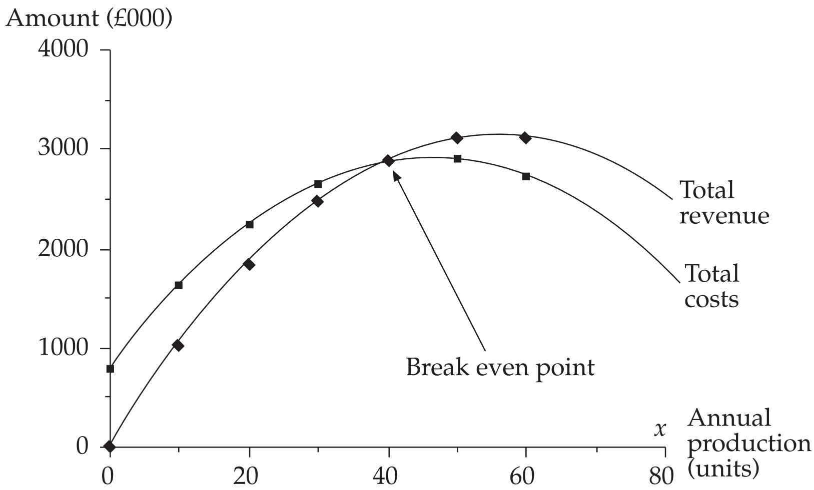
Therefore, $C_{max} = C(\text{at } x=46) = 92(46) - 46^2 + 800 = 2916$ (£000).

(e) The calculations for the plots of the two graphs are tabulated below.

x	0	10	20	30	40	50	60
$-x^2$	0	–100	–400	–900	–1600	–2500	–3600
$92x$	0	920	1840	2760	3680	4600	5520
800	800	800	800	800	800	800	800
C	800	1620	2240	2660	2880	2900	2720
$-x^2$	0	–100	–400	–900	–1600	–2500	–3600
$112x$	0	1120	2240	3360	4480	5600	6720
R	0	1020	1840	2460	2880	3100	3120

The two graphs are plotted in the figure on the following page.

From the graphs, it is clear that, since costs are falling while revenue is still increasing between the break-even production point and the maximum production point, the profit is increasing. Hence, maximum profit is obtained at maximum production.



Question 7

(a) Put x as the old price of the ticket. Then the number of tickets which could be purchased previously is $\frac{2850}{x}$ and the number which could be purchased after the

£6 price increase is $\frac{2850}{x+6}$

The price increase results in a reduction of 36 in the number of tickets which can be purchased. Therefore:

$$\begin{aligned}
\frac{2850}{x} &= \frac{2850}{x+6} + 36 \\
2850(x+6) &= 2850x + 36x(x+6) \\
2850x + 17100 &= 2850x + 36x^2 + 216x \\
0 &= 36x^2 + 216x - 17100 \\
0 &= x^2 + 6x - 475
\end{aligned}$$

Thus $x = -25$ (not feasible) or $x = 19$

The percentage increase in price is therefore $\frac{6}{19} \times 100 = 31.58\%$

(b) Quantity sold is $\frac{100}{p^2}$, price (P) is £p and cost (C) is 15p per toy.

$$\begin{aligned}
\text{Now, profit} &= \text{quantity sold} \times (\text{price} - \text{cost}) \\
&= \frac{100}{p^2} \times (P - C) \\
&= 100p^{-1} - 15p^{-2}
\end{aligned}$$

(i) $\frac{d\text{Profit}}{dx} = 100p^{-2} + 2 \times 15p^{-3}$

But the price at which profit is maximised is obtained by solving

$$\frac{d\text{Profit}}{dx} = 0$$

That is, $0 = (2)15p^{-3} - 100p^{-2}$

$0 = 30p^{-1} - 100$ giving $\frac{30}{p} = 100.$

Therefore, $p = 0.30$

(ii) Given that profit is maximised at a price of 30p, the maximum profit is given by $\frac{100}{p} - \frac{15}{p^2} = £166.67$

(iii) The quantity of toys sold at this level is $\frac{100}{0.3^2} = 1111.$

Question 8

(a) $TC = \int(x^2 - 28x + 211)dx = \frac{x^3}{3} - 14x^2 + 211x + c$

When $x = 0$, $TC = c$. and from the question when $x = 0$, $TC = 10$, so $c = 10$.

Therefore: $TC = \frac{x^3}{3} - 14x^2 + 211x + 10$

(b) $TR = (200 - 8x)x = 200x - 8x^2$

(c) Profit, $P = TR - TC = 200x - 8x^2 - \left(\frac{x^3}{3} - 14x^2 + 211x + 10\right) = \frac{x^3}{3} + 6x^2 - 11x - 10$

$$\frac{dP}{dx} = -x^2 + 12x - 11 = 0 \text{ for critical values.}$$

$-x^2 + 12x - 11 = 0 = x^2 - 12x + 11$

$(x - 11)(x - 1) = 0$

$x = 1$ or 11 (can also use the quadratic formula)

$\frac{d^2R}{dx^2} = -2x + 12$ and when x=1, $\frac{d^2R}{dx^2} = 10$ (minimum),

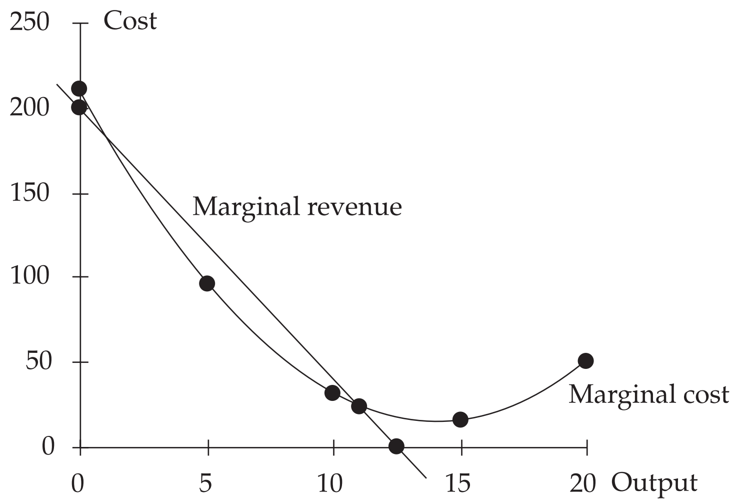
when x=11, $\frac{d^2R}{dx^2} = -10$ (maximum)

Therefore profit is maximised when output is 11.

(d) MR $= \frac{dTR}{dx} = 200 - 16x$. When x=0, MR=200. When MR=0, x=12.5.

When x=0, MC=211.

(e)



Question 9

(a) (i)

Given $C = aQ^2 - bQ + c$ and substituting for the information given, we have:

	$2900 = 100a - 10b$	 1
	$800 = 1600a - 40b + c$	 2
	$2000 = 10000a - 100b + c$	 3
1 – 2 :	$2100 = -1500a + 30b$	 4
3 – 2 :	$1200 = 8400a + 60b$	 5
4 × 2 :	$4200 = -3000a + 60b$	 6
5 + 6 :	$540000 = 5400a$. Therefore: $a = 1$.	
Substitute for a in 4 :	$2100 = -1500 + 30b$ Therefore: $b = 120$.	
Substitute for a, b in 1 :	$2900 = 100 - 1200 + c$ Therefore $c = 4000$	
Check for a,b and c in 2 :	$800 = 1600 - 4800 + 4000$. OK!	

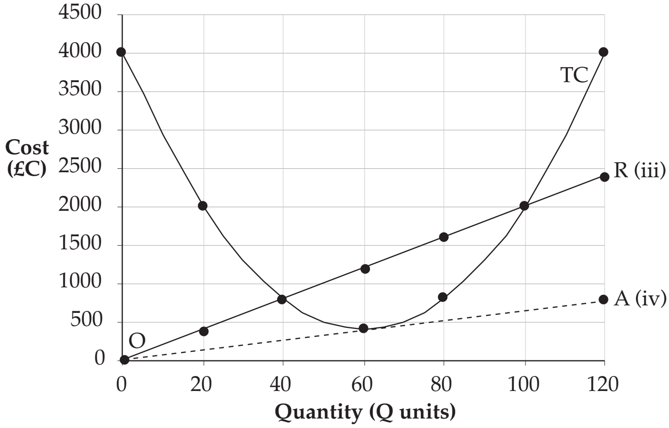
Thus: $C = Q^2 - 120Q + 4000$

(ii) The following table shows the values calculated for Total Costs (TC) for Q in the range [0,120] and the subsequent graph of TC plotted. It is easily seen that the quantity that minimises Total Costs (£400) is Q=60.

(iii) The Revenue Function (R) is shown tabulated and then plotted on the graph. The profit range required is such that $R > TC$ and the graph shows this is true for Q between 40 and 100.

(iv) If we call the minimum price MP, then we require a separate extra Revenue Function such that R = MP × Q will satisfy the relationship that it minimises Total Costs and the dotted line that does this is shown on the graph. The gradient of this line (ie MP) can be calculated as the gradient at point A which is (approximately) 800/120 = 6.6.

Q	TC	R
0	4000	0
20	2000	400
40	800	800
60	400	1200
80	800	1600
100	2000	2000
120	4000	2400



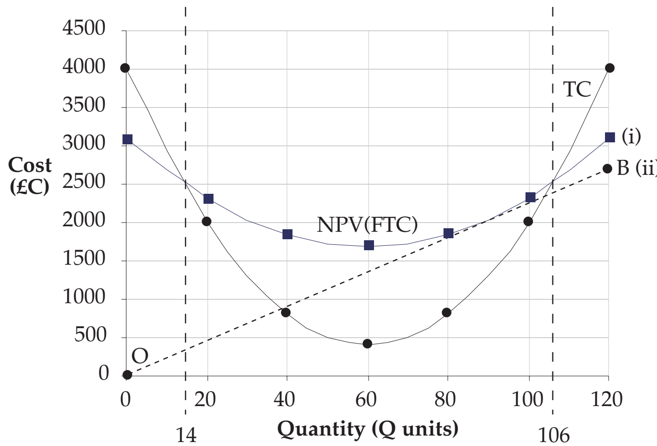
(b) Future Total Cost (FTC) = $Q^2 - 120Q + 8000$

The Net Present Value of this = NPV(FTC) = $\dfrac{Q^2 - 120Q + 8000}{(1.1)^{10}}$

= 0.38554 × NPV(FTC) from discount tables.

(i) These values are shown tabulated below and then plotted as shown. The production quantities for which real costs are the same are seen to be in the range [14, 106]

Q	FTC	NPV(FTC)
0	8000	3084
20	6000	2313
40	4800	1851
60	4400	1696
80	4800	1851
100	6000	2313
120	8000	3084



(ii) The minimum price that the product must be sold at in 10 years can be calculated by finding the gradient at point B = $\dfrac{2700}{120}$ = 22.5 (approx) and then inflating this to $22.5 \times (1.1)^{10} = 58.36$.

Part 8

Question 1

Here, Pr(Female) = Pr(F) = 0.65 and Pr(Male) = Pr(M) = 0.35.

(i) Pr(all female) = Pr(FFF) = $(0.65)^3$ = 0.275 (3D)

(ii) Pr(all male) = Pr(MMM) = $(0.35)^3$ = 0.043 (3D)

(iii) Pr(at least one male) = 1 – Pr(none male)
= 1 – Pr(all female)
= 1 – 0.275 [from (i) above]
= 0.725 (3D)

Question 2

Pr(system operates properly)
= Pr(at least one of A or B functions and at least one of C or D functions)
= Pr(at least one of A or B functions) × Pr(at least one of C or D functions)
= [1 – Pr(both A and B fail)] × [1 – Pr(both B and C fail)]
= [1 – (0.1)(0.1)] × [1 – (0.1)(0.1)] = [1 – 0.01] × [1 – 0.01]
= [0.99] × [0.99] = 0.9801

Question 3

(a) Pr(obtaining both contracts) = Pr(obtaining A and obtaining B)
= Pr(obtaining A) × Pr(obtaining B)
$= \frac{1}{3} \times \frac{1}{4}$
$= \frac{1}{12}$ = 0.083 (3D)

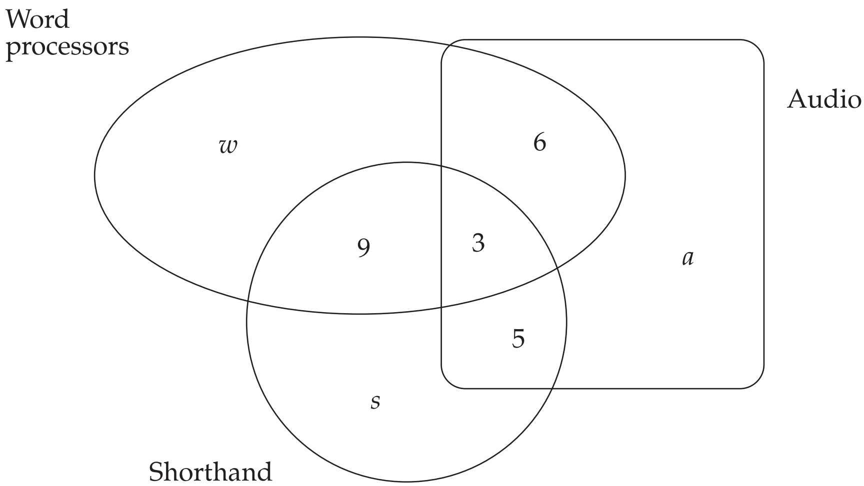
(b) (i) The Venn diagram is shown in the figure on the following page.

(ii) Number of typists who can use word processors
= n(W) = 30 = w + 6 + 3 + 9.
Thus, w=12. Similarly, n(A) = 25 = a + 5 + 3 + 6.
So that, a=11. And n(S) = 28 = s + 9 + 3 + 5. Therefore, s=11.
Therefore, the total number of typists is the sum of the number of elements in the seven distinct areas in Figure 1 = 9+3+5+6+12+11+11 = 57.

(iii) The number of typists with just one skill is a+w+s = 34.

Question 4

(a) Two events are statistically independent if the occurrence (or not) of one in no way affects the occurrence (or not) of the other.



(b) There are two factors involved here, minor accidents (yes/no) and safety instructions (yes/no). Using a numeric approach for the solution of the problem means evaluating the number of men that fall into each one of the four categories defined (see table).

Total of men who had minor accidents = 1% of 10,000 = 100. Thus, 40% of 100 = 40 had safety instructions. Also, 90% of the 10,000 = 9000 had safety instructions. Using these figures, the table below can be filled as follows:

		Minor accidents		
		Yes	No	Total
Safety Instructions	Yes	40	960	1000
	No	60	8940	9000
	Total	100	9900	10000

(i) Pr(No minor accidents / no safety instructions) = 0.993 (3D)

(ii) Pr(No minor accidents / safety instructions) = 0.96

(c) If there are x winning tickets out of 100, then:

Pr(win / x tickets) = $\frac{x}{100}$ with x possible between 1 and 15.

But $\Pr(x{=}1) = \Pr(x{=}2) = \ldots = \Pr(x{=}15) = \frac{1}{15}$

$$\begin{aligned}\text{Therefore, }\Pr(\text{win}) &= \Sigma\,[\,\Pr(x).\Pr(\text{win}/x)\,]\\ &= \Pr(x{=}1).\Pr(\text{win}/x{=}1) + \Pr(x{=}2).\Pr(\text{win}/x{=}2) + \ldots\\ &\qquad \ldots + \Pr(x{=}15).\Pr(\text{win}/x{=}15)\\ &= \frac{1}{15}\times\frac{1}{100} + \frac{1}{15}\times\frac{2}{100} + \ \ldots\ + \frac{1}{15}\times\frac{15}{100}\\ &= \frac{1}{1500}(1 + 2 + \ \ldots + 15) = \frac{120}{1500} = 0.08\end{aligned}$$

Question 5

If advertising method A is used in the next period, the expected weekly sales can be calculated:

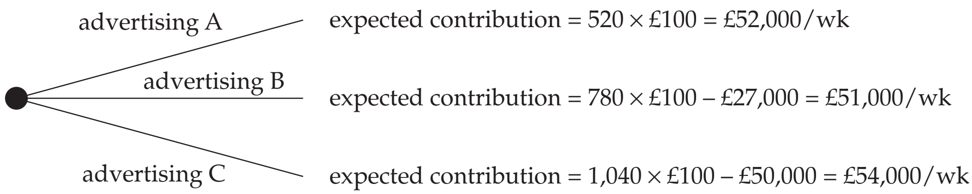
week sales, x units (mid-points)	probability, p	$x.p$
100	0	0
300	0.3	90
500	0.4	200
700	0.2	140
900	0.1	90
		520

Hence the expected weekly sales are 520 units.

If advertising B is used, this would increase expected weekly sales by 50%, to 780 units.

If advertising C is used,this would increase expected weekly sales by 100%, to 1,040 units.

The decision tree showing the options open to the company in the next period is:



(**Note**: The contribution of £100/unit is based on using A. The figures of £27,000 and £50,000 in the above are therefore the additional advertising expenditure, above that of A.)

Thus based on expected values, the best option is to use advertising C.

The expected profit from A is a more reliable estimate as it is based on past experiance, but even this depends on the assumption that what has happened in the past will happen again during the next period. The expected profits for B and C are more speculative as they are based on estimates (possibly subjective) of the affects of different, new types of advertising on sales.

All other things being equal, the expected profits for B and C are equally unreliable, and so C would seem to be a better option than B. It is far more debatable whether C is indeed a 'better' option than A: all the expected profits are so close that only a small variation in any of the estimates could alter their relative values. For example, if C's sales are only 2% down on those expected, the company will be worse off than sticking with A. This, combined with the relatively higher reliability of the expected value for A, would lead many organisations to choose A. Only the more risk-seeking would choose C.

Question 6

(a) To find the expected (mean) sales.

Number of loaves sold (x)	0	100	200	300	400	Total
Number of days (f)	10	60	60	50	20	200
Probability (p)	0.05	0.30	0.30	0.25	0.10	

Expected number of loaves sold $= \sum$ (number of loaves × probability)

$\sum px = 0(0.05)+100(0.30)+200(0.30)+300(0.25)+400(0.10) = 205.$

(b)

PROFIT		Production			
		100	200	300	400
Sales	0	–10	–20	–30	–40
	100	20	10	0	–10
	200	20	40	30	20
	300	20	40	60	50
	400	20	40	60	80

(c) Expected profit = actual profit × probability. The following table shows expected profit. i.e. each profit value from the table in (b) multiplied by the probability of the respective sales level given.

EXPECTED PROFIT TABLE

			Production			
			100	200	300	400
Sales	Value	Pr				
	0	0.05	–0.5	–1	–1.5	–2
	100	0.30	6	3	0	–3
	200	0.30	6	12	9	6
	300	0.25	5	10	15	12.5
	400	0.10	2	4	6	8
Expected profit			18.5	28	28.5	21.5

(d) The optimum level of production is 300 loaves per day, since this has the highest expected profit of £28.50 per day.

Question 7

EV = expected value

At C: EV (shelve) = –4
EV (market) = (0.8 × 10) + (0.2 × 20) = 12
∴ market

At D: EV (shelve) = –4
EV (market) = (0.5 × 0) + (0.5 × 5) = 2.5
∴ market

At E: EV (shelve) = –4
EV (market) = (–4 × 0.1) + (–3 × 0.9) = –3.1
∴ market

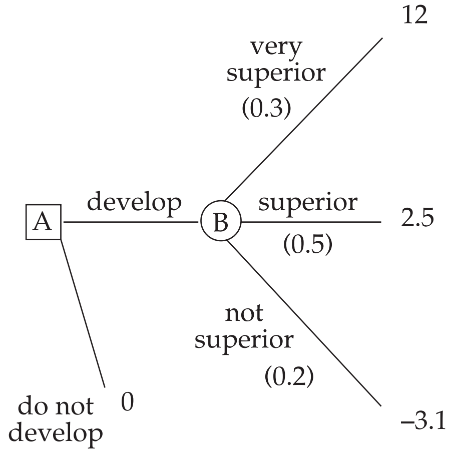
The ‘rolled-back’ tree is shown on the opposite page.

At A: EV (do not develop) = 0
EV (develop) = (0.3 × 12) + (0.5 × 2.5) +(1.2 × – 3.1) = 4.23

∴ The company should develop.

All of the above decisions have been based on the criterion of maximising expected values (of profits). At F and G, the decisions are clear-cut, since the choices in both cases are between one option which at least breaks even and one which loses £4m. The decision is also clear-cut at E, because one option guarantees losing £4m, while the other can lose at most £4m, but might incur a reduced loss of £3m.

At A, the decision is more debatable, as the 'develop' option could lead to a loss (at E), while 'not develop' guarantees no loss. The chances of the loss are, however, small (20%) and the potential gains are large, and so many companies would be willing to take the risk of developing.



Question 8

(a) For product A

Probability of low demand = 1 – 0.7 = 0.3

Expected profit £ =0.2 × 2 + 0.5 × 1.5 + 0.3 × 0.75 = 0.4 + 0.75 + 0.225 = 1.375

For product B

Probability of medium demand = 1 – 0.4 = 0.6

Expected profit £ =0.3 × 1.5 + 0.6 × 1 + 0.1 × 1.5 = 0.45+ 0.6 + 0.05 = 1.1

The shop should display product A.

(b) i. A random sample is one where each member of the population has an equal chance of being selected for the sample. The appropriate measures for such a sample are the mean and standard deviation.

ii. In a quota sample, all the members of the population do not have an equal chance of being selected for the sample. An interviewer is required to select a number of interviewees who may be required to have certain characteristics. As the selection is not random the appropriate statistical measures are the median and semi-interquartile range.

iii. A cluster is one where the population is divided into sub-groups which may represent geographical areas. A sample is then chosen at random within each cluster. As the sampling is random the mean and standard deviation may be used. Alternatively a cluster may be chosen at random and all the members of the cluster interviewed, again the mean and standard deviation are suitable measures of location and dispersion.

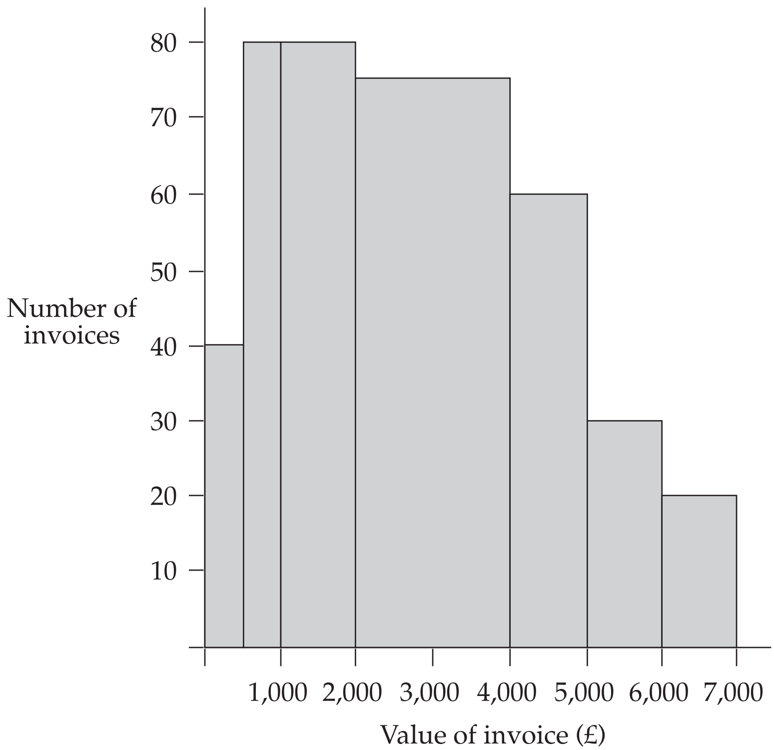
Part 9

Question 1

(a) The bar heights of the classes that have a width different to the standard (taken here as 1,000) need to be adjusted.

Lower limit	Upper limit	Act f	Bar height
0	500	20	2×20=40
500	1,000	40	2×40=80
1,000	2,000	80	80
2,000	4,000	150	0.5×150=75
4,000	5,000	60	60
5,000	6,000	30	30
6,000	7,000	20	20

The chart is shown below.



(b) Using the mid-points of each class as a representative figure

x £	f	fx
250	20	5,000
750	40	30,000
1,500	80	120,000
3,000	150	450,000
4,500	60	270,000
5,500	30	165,000
6,500	20	130,000
	400	1,170,000

The mean value is $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{1{,}170{,}000}{400} = £2{,}925.$

Note that s = standard deviation = £1,600 (given)

Thus a 90% CI is: $\bar{x} = 1.64\dfrac{s}{\sqrt{n}} = 1.64\dfrac{1600}{\sqrt{400}} = 2925 \pm 131.2 = (27938.8, 3056.2)$

(c) If 20 invoices (out of 400) contain errors then $p = \dfrac{1}{20} = 0.05$

Thus a 95% confidence interval is:

$$p \pm 1.96\sqrt{\frac{p(1-p)}{n}} = 0.05 \pm 1.96\sqrt{\frac{(0.05)(0.95)}{400}}$$

$$= 0.05 \pm 0.02$$

$$= (0.03, 0.07)$$

Question 2

(a) The lifetimes (L, say) are distributed Normally with mean, m=100 and standard deviation, s unknown.

(i) If 90% of the batteries last at least 40 hours, we have: $\Pr(L>40) = 0.9$.

Standardising gives $\Pr\left(Z > \dfrac{40-100}{s}\right) = 0.9$. i.e. $\Pr\left(Z > \dfrac{-60}{s}\right) = 0.9$

Now, from Standard Normal tables, a table probability of 0.9 yields $Z = 1.28$.

Hence, $\dfrac{-60}{s} = 1.28$, giving $s = 46.88$ (2D).

(ii) We require Pr(L<70).

Standardising gives: $\Pr\left(Z < \dfrac{70-100}{46.88}\right) = \Pr\,(Z < -0.64)$

$= 1 - 0.7389$ [from tables]

$= 0.2611$

Thus, 26% of batteries will not last 70 hours.

(b) This is a binomial situation with:

n = size of sample = 5 and p = Pr(defective) = 0.1.

(i) We require Pr(at least 3 defectives) = Pr(3 or 4 or 5 defectives)

= Pr(3) + Pr(4) + Pr(5)

= 5C3(0.1)3(0.9)2 + 5C4(0.1)4(0.9)1 + 5C5(0.1)5(0.9)2

= 10(0.1)3(0.9)2 + 5(0.1)4(0.9)1 + (0.1)5

= 0.0081 + 0.00045 + 0.00001

= 0.00856

= 0.01 (2D).

(ii) The result of (i) shows that only one time in a hundred should there be 3 or more defectives out of a total of 5 calculators examined. The fact that this has happened after one sample (i.e. 1 out of 1) must throw suspicion on the original assumption that only 10% of calculators are defective. The conclusion would be that the process is not working satisfactorily.

Question 3

(a) A has a Normal distribution with m=1000 kgs and s =100 kgs; B has a Normal distribution with m =900 kgs and s=50 kgs; the breaking strength must be at least 750 kgs (given).

Thus we need to find those ropes which have the greatest probability of a breaking strength greater than 750 kgs.

$\Pr(A>750) = \Pr\left(Z > \dfrac{750-1000}{100}\right)$ [standardising] = Pr(Z>–2.5) = 0.9938 [tables]

$\Pr(B>750) = \Pr\left(Z > \dfrac{750-900}{50}\right)$ [standardising] = Pr(Z>–3) = 0.9987 [tables]

Thus, supplier B's ropes should be bought.

(b) This is a binomial situation with n=50 and p=Pr(defective)=0.01.

(i) Using the binomial distribution:

Pr(0 defectives) = ${}^{50}C_0(0.01)^0(0.99)^{50} = (0.99)^{50} = 0.605$ (3D).

(ii) Using the poisson distribution, the mean is calculated as the mean of the binomial.

i.e. $m = np = 50(0.01) = 0.5$. Therefore, Pr(0 defectives) = $e^{-0.5} = 0.607$ (3D).

Question 4

(a) We are given a Normal distribution of claims, with $m = 200$ and $s = \sqrt{2500} = 50$

(i) Pr(claim<100) = $\Pr\left(Z > \frac{100-200}{50}\right)$ [standardising] = Pr(Z<–2) = 1 – 0.9772

[from tables] = 0.0228.

Thus, only about 2% of claims will be under £100.

(ii) Pr(claim<150) = $\Pr\left(Z > \frac{150-200}{50}\right)$ [standardising] = Pr(Z<–1) = 1 – 0.8413

[from tables] = 0.1587.

Thus, about 16% of claims will be under £150.

(iii) Pr(claim>350) = $\Pr\left(Z > \frac{350-200}{50}\right)$ [standardising] = Pr(Z>–3) = 1 – 0.9987

[from tables] = 0.0013.

That is, only about 0.1% of claims will be over £350.

(b) This is a binomial situation with p = Pr(passenger turns up) = 0.9 and n = 290 (number of bookings taken). Using the Normal approximation, we have:
m = mean = $n.p$ = 290(0.9) = 261
s = standard deviation = $\sqrt{[n.p.(1-p)]} = \sqrt{[290(0.9)(0.1)]}$ = 5.11 (2D)
We need the probability that the number of passengers who turn up (P, say) will exceed 275 (i.e. 276 or 277 or ... etc), where it should be carefully noted that 276 for a binomial is the equivalent of the range 275.5 to 276.5 for a Normal distribution.

Thus, we require: Pr(P>275.5) = $\Pr\left(Z > \frac{275.5-261}{5.11}\right)$ [standardising]

= Pr(Z>2.84)

= 1 – 0.9977 [from tables] = 0.0023.

Question 5

(a) We are given a poisson situation with mean, m=2 (demands for a coach each day). NOTE: Proportions and probabilities are identical concepts.

(i) Pr(neither coach used) = Pr(0 demands) = e^{-2} = 0.1353.

(ii) At least 1 demand refused means that at least 3 demands have been made (since there are 2 coaches available)

Thus we require Pr(at least 3 demands)

= 1 – Pr(0 or 1 or 2 demands)

= 1 – [Pr(0) + Pr(1) + Pr(2)]

$= \left[0.1353 + 2.e^{-1} + \frac{2^2}{2}.e^{-2}\right]$

= 1 – [0.1353 + 0.2706 + 0.2706]

= 0.324 (3D).

(iii) Consider coach A. If no coaches are demanded, then coach A will not be used. This will happen proportion Pr(0) = 0.1353 of the time. If 1 coach is demanded, there is only a 50% chance that it will be A that is not used. This will happen with probability: 0.5× Pr(1) = 0.5× 0.2706 = 0.1353. Thus the total proportion of times that coach A will not be used is: 0.1353 + 0.1353 = 0.271 (3D).

(b) The lengths (L, say) are given as normal with mean, m=20.02 and standard deviation, s=0.05 cm.

(i) Pr(L will be undersize) = Pr(L<19.9) = $\Pr\left(Z < \frac{19.1 - 20.02}{0.05}\right)$ = Pr(Z<–2.4)
= 1 – 0.9918 [from tables] = 0.0082.

i.e. 0.8% of rods will be undersize.

(ii) Pr(L will be oversize) = Pr(L>20.1) = $\Pr\left(Z < \frac{20.1 - 20.02}{0.05}\right)$ = Pr(Z>1.6)
= 1 – 0.9452 [from tables] = 0.0548.

Thus, approximately 5.5% of rods will be oversize.

Question 6

(a) The Normal distribution is known as the distribution of 'natural phenomena' and is the most commonly occurring continuous distribution. Heights, weights, lengths and times commonly form Normal distributions, which are characterized by their 'bell-shaped' curves.

(b) Consumption (C, say) is given as a Normal distribution with m = 10,000 and s = 2000.

(i) Pr(C>13000) = $\Pr\left(Z < \frac{13000 - 10000}{2000}\right)$ [standardising]
= Pr(Z>1.5) = 1 – 0.9332 [from tables]
= 0.0668.

(ii) Pr(C<8000) = $\Pr\left(Z < \frac{8000 - 10000}{2000}\right)$ [standardising]
= Pr(Z<–1) = 1 – 0.8413 [from tables]
= 0.1587.

(iii) To find Pr(7500<C<14000) = 1 – Pr(C<7500) – Pr(C>14000).

But Pr(C<7500) = $\Pr\left(Z < \frac{7500 - 10000}{2000}\right)$ [standardising]
= Pr(Z<–1.25) = 1 – 0.8944 [from tables]
= 0.1056

Also Pr(C>14000) = $\Pr\left(Z < \frac{14000 - 10000}{2000}\right)$ [standardising]
= Pr(Z>2) = 1 – 0.9772 [from tables]
= 0.0228.

Therefore, Pr(7500<C<14000) = 1 – 0.1056 – 0.0228 = 0.8716.

Question 7

Putting S as the scores Normal variable, with mean 60 and standard deviation 12, we require:

$$\Pr(S > 75) = \Pr\left(Z < \frac{75-60}{60}\right) = \Pr(Z > 1.25) = 1 - \Pr(Z < 1.25) = 1 - 0.8944 = 0.1056$$

Question 8

(a) Mean in sample of 10 = 1.6 Therefore p(box underweight) = 1.6/10 = 0.16

(b) We have n=6 and p(underweight)=0.16 and a binomial situation.

(i) $p(0) = {}^6C_0.(0.16)^0(0.84)^6 = 0.3513$

(ii) $p(2) = {}^6C_2.(0.16)^2(0.84)^4 = 15(0.056)(0.4979) = 0.1911$

Also $p(1) = {}^6C_1.(0.16)^1(0.84)^5 = 0.4015$

(iii) p(at least 3) = 1 – p(0, 1 or 2)
= 1 – (0.3513 + 0.1911 + 0.4015)
= 0.0561

(c) Since n=100, we can approximate to a Normal distribution with mean = np = 100(0.16) = 16 and sd = √100(0.16)(0.84) = 3.67

(i) p(<10) = p(<9.5) with continuity correction

Standardising 9.5 gives $z = \frac{9.5 - 16}{3.67} = -1.77$

Thus: p(<10) = p(Z<–1.77)
= 1 – p(Z<1.77)
= 1 – 0.9938
= 0.01

(ii) p(>28) = p(>28.5) with continuity correction

Standardising 28.5 gives $z = \frac{28.5 - 16}{3.67} = 3.41$

Thus: p(>28) = p(Z>3.41)
= 1 – p(Z<3.41)
= 1 – 1
= 0

(iii) p(between 16 and 24) = p(<24) – p(<16)
= p(<23.5) – p(15.5) with continuity correction

Standardising 23.5 gives $z = \frac{23.5 - 16}{3.67} = 2.04$

Standardising 15.5 gives $z = \frac{15.5 - 16}{3.67} = -0.14$

p(between 16 and 24) = p(Z<2.04) – p(Z<–0.14)
= p(Z<2.04) – [1 – p(Z<0.14)]
= p(Z<2.04) + p(Z<0.14) – 1
= 0.98 + 0.54 – 1
= 0.52

Part 10

Question 1

a) i. A TV spot will generate 1000 extra sales at a gross profit of £10/unit, and will cost £5,000.

The contribution is thus: 1,000 × 10 – 5,000 = £5,000

ii. In the same way, the contribution of a newspaper advertisement is 400 × 10 – 2,000 = £2,000

b) Suppose the company buys x TV spots and y newspaper advertisements.

The objective is to maximise contribution, $z = 5{,}000x + 2{,}000y$ (£).

The constraints are: advertising budget:

$5{,}000x + 2{,}000y \leq 100{,}000$ or $5x + 2y \leq 100$ (1)

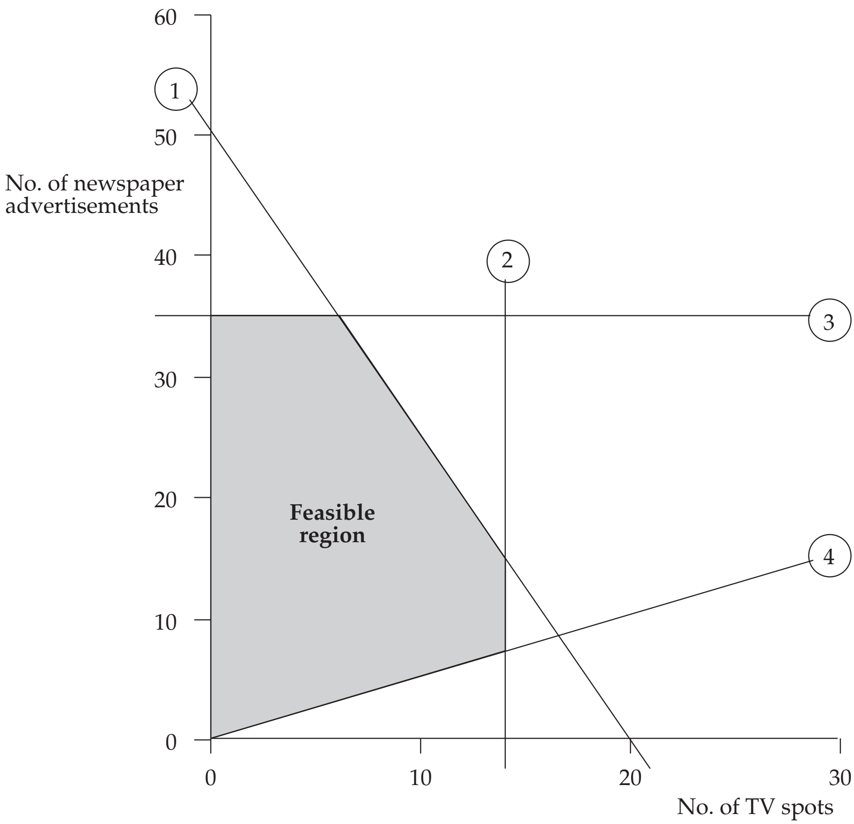
maximum to be spent on each mode:

$5{,}000x \leq 70{,}000$ or $x \leq 14$ (2)

$2{,}000y \leq 70{,}000$ or $y \leq 35$ (3)

for marketing balance: $y \geq \frac{1}{2}x$ or $2y \geq x$ (4)

c)



Vertex B will clearly represent an advertising mix which will generate a higher contribution than that of A. Similarly, mix C will generate a higher contribution than mix D. Only these two vertices of the feasible area are therefore considered.

Vertex	(x,y)	$z = 5{,}000x + 2{,}000y$ (£)
B	(6,35)	100,000
C	(14,15)	100,000

Thus the maximum contribution which can be generated by these promotions is £100,000. This can be achieved by any mix on the line (1) of the graph, between B and C, provided x and y are whole numbers.

These are:

x (number of TV spots)	y (number of newspaper advertisements)
6	35
8	30
10	25
12	20
14	15

It can be seen that there are *five* different ways of achieving the maximum contribution. This gives the company added flexibility, in that it can choose one from the five which gives extra benefits. For example, if the company felt that television advertising gave a high profile to all its products (not just Brand X), then it could choose the last of the combinations listed, and still maximise *X*'s contribution.

Question 2

The purpose of this question is to test the candidates knowledge of probability and of matrix notation.

a) i. Let F represent the event of failing the test and A the event of being appointed.

Then P(F) = 0.8 hence P (not F) = 1 – P(F) = 1 – 0.8 = 0.2

From the question P(A/not F) = 0.3 and P(A / F) = 0.1

The probability of being appointed, P(A), is

P(A) = P(A/F)P(F) + P(A/not F)P(not F) = 0.1 × 0.8 + 0.3 × 0.2 = 0.14

ii. P(not A) = 1 – P(A) = 1 – 0.14 = 0.86

P(not A/F) = 1 – P(A/not F) = 1 – 0.3 = 0.7

P(not A intersection not F) = P(not A/not F) × P(not F) = 0.7 × 0.2 = 0.14

P(not F/not A) = P(not A intersection not F)/P(not A)

$$= \frac{0.14}{0.86} = \frac{7}{43} = 0.163$$

iii. Probability of being appointed for all applicants

= probability of being selected for first interview × probability of being selected = 0.15 × 0.14 = 0.021

b) i. $x(1) + x(2) + 2x(3) = 12{,}000$

$$x(1) + 2x(2) + 4x(3) = 22{,}000$$
$$2x(1) + 2x(2) + 3x(3) = 19{,}500$$

The above equations in matrix format are: $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 2 & 2 & 3 \end{bmatrix} * \begin{bmatrix} x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 12{,}000 \\ 22{,}000 \\ 19{,}500 \end{bmatrix}$

ii. Profit (£) = $4 \times 2{,}000 + 4 \times 1{,}000 + 2 \times 4{,}500 = 21{,}000$

iii. Profit (£) = $4 \times 3{,}000 + 4 \times 1{,}500 + 2 \times 5{,}000 = 28{,}000$

Annual market value (£) = $28{,}000 \times 4 \times 12 = 112{,}000 \times 12 = 1{,}344{,}000$

Question 3

(a) The conditions under which production is possible are:

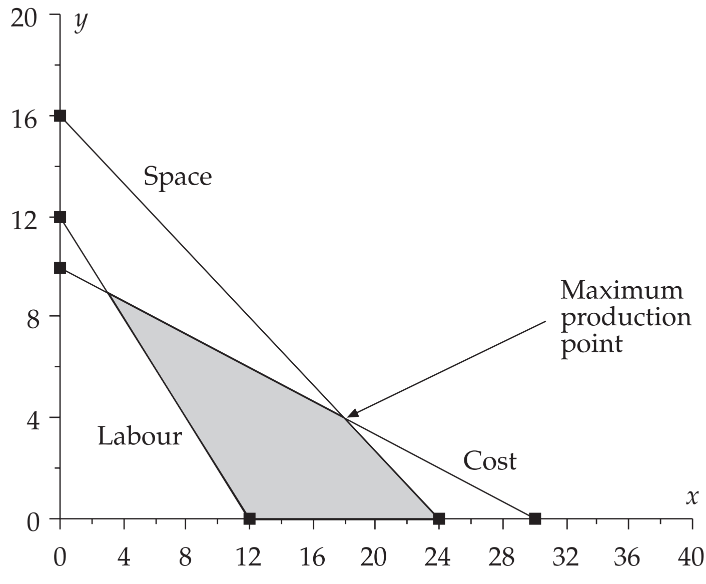
Labour: $2X + 2Y \geq 24$ or $X + Y \geq 12$

Cost: $4000X + 12000Y \leq 120000$ or $X + 3Y \leq 30$

Space: $20X + 30Y \leq 480$ or $2X + 3Y \leq 48$

The (production) function to be maximised is $150X + 300Y$.

(b) See the figure below.



(c) From the graph, the maximum production point is x=18, y=4.
That is, 18 X presses and 4 Y presses should be bought.

(d) Maximum production is 150(18) + 300(4) = 3900 sheets/minute.
Total cost of presses is 18(4000) + 4(12000) = £120,000.

Question 4

(a) (i) If P = 192 – 28r + r^2, then the break even point is where P = 0.
i.e. where $r^2 - 28r + 192 = 0$.
Solution by factorization: (r–16)(r–12)=0. Therefore r=12 or r=16.
Solution by formula: (with a=1, b=–28 and c=192).

$$r = \frac{28 \pm \sqrt{28^2 - 4(1)(192)}}{2(1)} = \frac{28 \pm \sqrt{16}}{2} = \frac{28+4}{2} \text{ or } \frac{28-4}{2} = 16 \text{ or } 12$$

(ii) Consider a graph of $P = 192 - 28r + r^2$. It must cross the r-axis at the two points r=12 and r=16. But P is a 'U-shaped' parabola, which means that P must be negative between the values r=12 and r=16 and positive for all other values of r. Thus, the particular project in question makes a loss if the discount rate is between 12% and 16% and a profit for all other values.

(b) (i) If C is the cost matrix and D is the demand matrix, then:

$$C = \begin{array}{c} \\ \text{Shirts} \\ \text{Shorts} \\ \text{Socks} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 5.75 & 6.25 \\ 3.99 & 4.48 \\ 1.85 & 1.97 \end{bmatrix} \end{array} \text{ and } D = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{X} & \text{Y} & \text{Z} \end{array} \\ \begin{bmatrix} 36 & 24 & 60 \\ 48 & 72 & 0 \end{bmatrix} \end{array}$$

(ii) $$C{*}D = \begin{bmatrix} 5.75 & 6.25 \\ 3.99 & 4.48 \\ 1.85 & 1.97 \end{bmatrix} * \begin{bmatrix} 36 & 24 & 60 \\ 48 & 72 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ \text{Shirts} \\ \text{Shorts} \\ \text{Socks} \end{array} \begin{array}{c} \begin{array}{ccc} \text{X} & \text{Y} & \text{Z} \end{array} \\ \begin{bmatrix} 507 & 588 & 345 \\ 358.68 & 418.32 & 239.40 \\ 161.16 & 186.24 & 111.00 \end{bmatrix} \end{array}$$

(c) The new demand matrix is:

$$D = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{X} & \text{Y} \end{array} \\ \begin{bmatrix} 36 & 24 \\ 48 & 72 \end{bmatrix} \end{array}$$

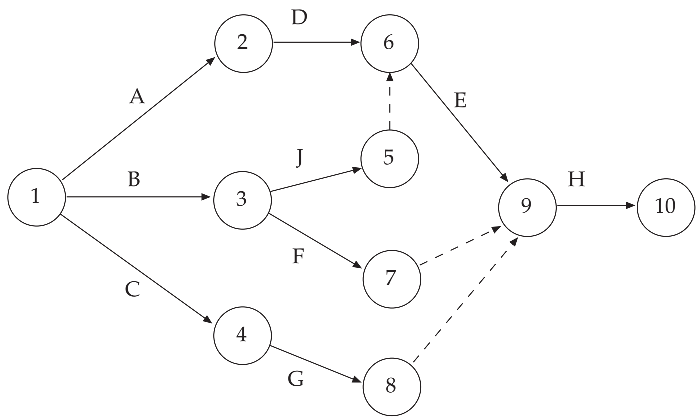
and its inverse can be calculated as follows:

$$D^{-1} = \frac{1}{(36)(72)-(48)(24)} \begin{bmatrix} 72 & -24 \\ -48 & 36 \end{bmatrix} = \begin{bmatrix} 72/1440 & -24/1440 \\ -48/1440 & 36/1440 \end{bmatrix}$$

$$= \begin{bmatrix} 0.05 & -0.017 \\ -0.033 & 0.025 \end{bmatrix}$$

Question 5

a)



b)

	Activity	Duration	Earliest start	Earliest finish	Latest start	Latest finish	Total Float
A	1–2	3	0	3	2	5	2
B	1–3	3	0	3	2	5	2
C	1–4	7	0	7	0	7	0
D	2–6	1	3	4	5	6	2
J	3–5	1	3	4	5	6	2
F	3–7	2	3	5	6	8	3
G	4–8	1	7	8	7	8	0
	5–6	0	4	4	6	6	2
E	6–9	2	4	6	6	8	2
	7–9	0	5	5	8	8	3
	8–9	0	8	8	8	8	0
H	9–10	1	8	9	8	9	0

c) The critical path is C–G–H (Alternatively 1–4–8–9–10). This path determines the minimum project completion time (9 days). All activities on the critical path must be completed without delay if the project is to be completed in 9 days.

Question 6

a) If the company buys x tables of type X and y tables of type Y, then the objective function is $x + y$, which needs to be minimised.

b) The constraints are given as follows:

Money: $40x + 30y \leq 24000$

Seating capacity: $4x + 2y \geq 1800$

Mixture of x and y: $x - y \leq 0$

c) The graphs of the constraints are shown in the figure, with the feasible region marked.

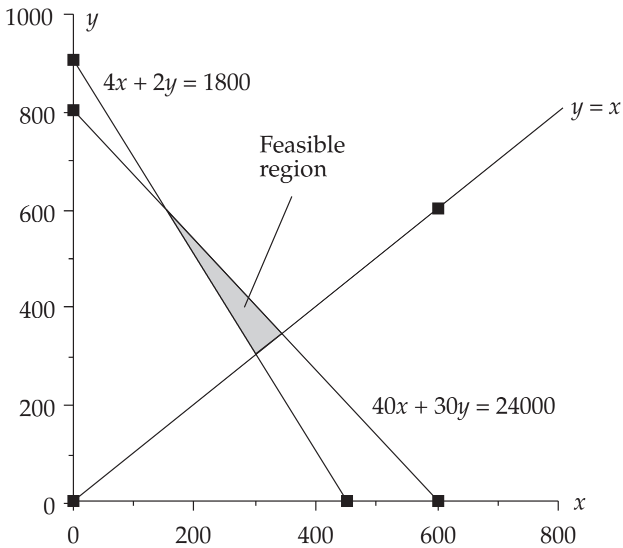
d) The optimum solution can be found by evaluating $x + y$ for each of the three vertices of the feasible region.

For the vertex at x=300, y=300, $x + y = 600$

For the vertex at x=150, y=600, $x + y = 750$

For the vertex at x=342.9, y=342.9, $x + y = 685.8$.

Thus the minimum total number of tables needed is 600, 300 of type X and 300 of type Y.



Question 7

(i) The matrix showing the pattern of retention and transfer from the first to the second month is:

$$S = \begin{array}{c} \\ BM1 \\ BM2 \\ BM3 \end{array} \begin{array}{c} \begin{array}{ccc} BM1 & BM2 & BM3 \end{array} \\ \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} \end{array}$$

(ii) The product of matrix S with itself is:

$$\begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} * \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} = \begin{bmatrix} 0.5450 & 0.2850 & 0.1700 \\ 0.3425 & 0.4875 & 0.1700 \\ 0.1125 & 0.2275 & 0.6600 \end{bmatrix}$$

(iii) The resulting matrix can be interpreted as follows.

Of the original customers who buy BM1, 54.5% will remain loyal to the brand in month 3, 28.5% will have switched to BM2 and 17% will have switched to BM3.

Of the original customers who buy BM2, 48.75% will remain loyal to the brand in month 3, 34.25% will have switched to BM1 and 17% will have switched to BM3.

Of the original customers who buy BM3, 66% will remain loyal to the brand in month 3, 11.25% will have switched to BM1 and 22.75% will have switched to BM2.

Question 8

(a) $C = D\frac{c_1}{q} + q\frac{c_2}{2}$; $\frac{dC}{dq} = -D\frac{c_1}{q^2} + \frac{c_2}{2} = 0$ for critical values gives $q = \sqrt{2c_1\frac{D}{c_2}}$

and $\frac{d^2C}{dq^2} = 2D\frac{c_1}{q^3}$

When q is the positive square root and as all other quantities are positive then the second derivative is positive values of q. This implies that there is a minimum in cost for positive values of q.

(b) Demand rate d = 1,000; Order cost, c_1= 10; Storage cost, c_2= 2.

$$\text{EOQ} = \sqrt{2c_1\frac{d}{c_2}}$$

(c)

Order size	Ordering cost £	Storage cost £	Total cost £
0	0	∞	∞
20	500	20	520
40	250	40	290
60	166.66	60	226.66
80	125	80	205
100	100	100	200
120	83.33	120	203.33
140	71.42	140	211.42
160	62.5	160	22.5

(d) Order cost, storage cost and total cost against order size. See Figure Q8(d).

(e) The model assumes that demand is regular, that stock delivery is instantaneous and that there is no chance of stockout.

Question 9

(a) Let X be the number of boxes of X produced and y be the number of boxes of Y produced.

The given restrictions (constraints) are tabulated below as linear inequalities.

Blending:	$3x + y < 900$
Baking:	$5x + 4y < 1800$
Packaging:	$x + 3y < 900$

We need to maximise the objective function, $x + 2y$.

(b) The constraints are graphed in Figure Q9(b).

The optimum solution can be read from the graph as x=164 and y=245. That is, 164 X chocolate bars and 245 Y chocolate bars should be produced in order to maximise contribution.

(c) The maximum contribution is obtained by substituting the optimal solutions into the objective function, $x + 2y$, to give 164 + 2(245) = 654. That is, the maximum contribution is £654.

Figure Q8(d)

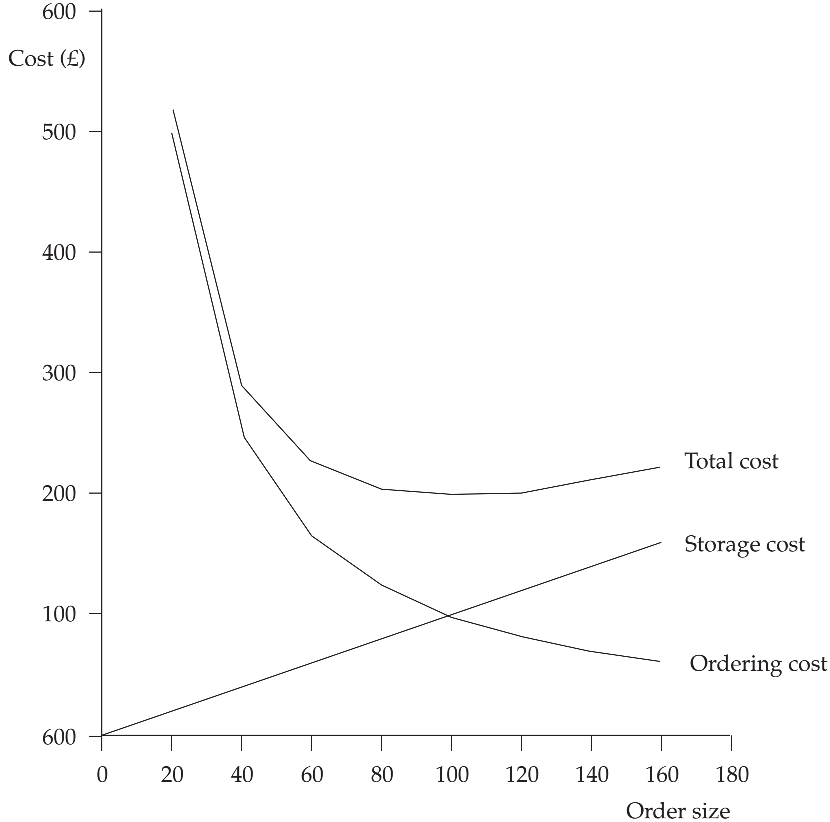
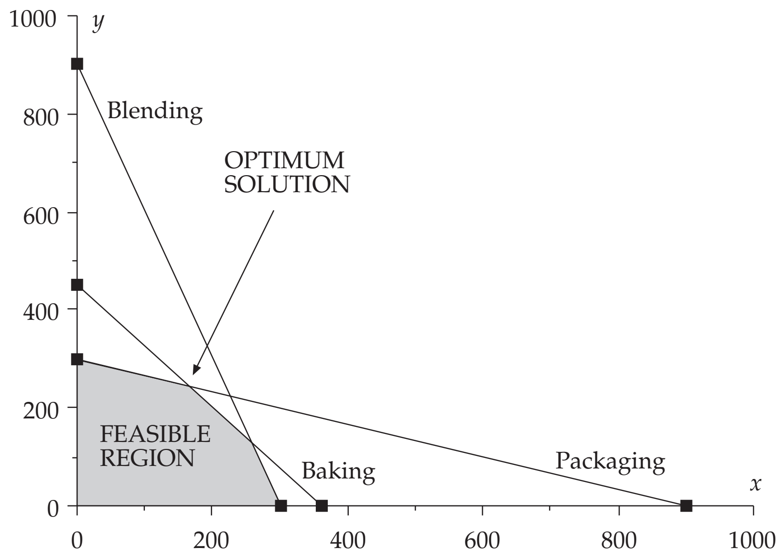


Figure Q9(b)



Appendices

1 Compounding and discounting tables

Range: n = 1 to 8, 1% to 12%

[C = multiplier for compounding; D = multiplier for discounting]

		n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1%	C	1.0100	1.0201	1.0303	1.0406	1.0510	1.0615	1.0721	1.0829
	D	0.9901	0.9803	0.9706	0.9610	0.9515	0.9420	0.9327	0.9235
2%	C	1.0200	1.0404	1.0612	1.0824	1.1041	1.1262	1.1487	1.1717
	D	0.9804	0.9612	0.9423	0.9238	0.9057	0.8880	0.8706	0.8535
3%	C	1.0300	1.0609	1.0927	1.1255	1.1593	1.1941	1.2299	1.2668
	D	0.9709	0.9426	0.9151	0.8885	0.8626	0.8375	0.8131	0.7894
4%	C	1.0400	1.0816	1.1249	1.1699	1.2167	1.2653	1.3159	1.3686
	D	0.9615	0.9246	0.8890	0.8548	0.8219	0.7903	0.7599	0.7307
5%	C	1.0500	1.1025	1.1576	1.2155	1.2763	1.3401	1.4071	1.4775
	D	0.9524	0.9070	0.8638	0.8227	0.7835	0.7462	0.7107	0.6768
6%	C	1.0600	1.1236	1.1910	1.2625	1.3382	1.4185	1.5036	1.5938
	D	0.9434	0.8900	0.8396	0.7921	0.7473	0.7050	0.6651	0.6274
7%	C	1.0700	1.1449	1.2250	1.3108	1.4026	1.5007	1.6058	1.7182
	D	0.9346	0.8734	0.8163	0.7629	0.7130	0.6663	0.6227	0.5820
8%	C	1.0800	1.1664	1.2597	1.3605	1.4693	1.5689	1.7138	1.8509
	D	0.9259	0.8573	0.7938	0.7350	0.6806	0.6302	0.5835	0.5403
9%	C	1.0900	1.1881	1.2950	1.4116	1.5386	1.6771	1.8280	1.9926
	D	0.9174	0.8417	0.7722	0.7084	0.6499	0.5963	0.5470	0.5019
10%	C	1.1000	1.2100	1.3310	1.4641	1.6105	1.7716	1.9487	2.1436
	D	0.9091	0.8264	0.7513	0.6830	0.6209	0.5645	0.5132	0.4665
11%	C	1.1100	1.2321	1.3676	1.5181	1.6851	1.8704	2.0762	2.3045
	D	0.9009	0.8116	0.7312	0.6587	0.5935	0.5346	0.4817	0.4339
12%	C	1.1200	1.2544	1.4049	1.5735	1.7623	1.9738	2.2107	2.4760
	D	0.8929	0.7972	0.7118	0.6355	0.5674	0.5066	0.4523	0.4039

Range: n = 1 to 8, 13% to 25%

[*C* = multiplier for compounding; *D* = multiplier for discounting]

		n=1	*n*=2	*n*=3	*n*=4	*n*=5	*n*=6	*n*=7	*n*=8
13%	C	1.1300	1.2769	1.4429	1.6305	1.8424	2.0820	2.3526	2.6584
	D	0.8850	0.7831	0.6931	0.6133	0.5428	0.4803	0.4251	0.3762
14%	C	1.1400	1.2996	1.4815	1.6890	1.9254	2.1950	2.5023	2.8526
	D	0.8772	0.7695	0.6750	0.5921	0.5194	0.4556	0.3996	0.3506
15%	C	1.1500	1.3225	1.5209	1.7490	2.0114	2.3131	2.6600	3.0590
	D	0.8696	0.7561	0.6575	0.5718	0.4972	0.4323	0.3759	0.3269
16%	C	1.1600	1.3456	1.5609	1.8106	2.1003	2.4364	2.8262	3.2784
	D	0.8621	0.7432	0.6407	0.5523	0.4761	0.4104	0.3538	0.3050
17%	C	1.1700	1.3689	1.6016	1.8739	2.1924	2.5652	3.0012	3.5115
	D	0.8547	0.7305	0.6244	0.5337	0.4561	0.3898	0.3332	0.2848
18%	C	1.1800	1.3924	1.6430	1.9388	2.2878	2.6996	3.1855	3.7589
	D	0.8475	0.7182	0.6086	0.5158	0.4371	0.3704	0.3139	0.2660
19%	C	1.1900	1.4161	1.6852	2.0053	2.3864	2.8398	3.3793	4.0214
	D	0.8403	0.7062	0.5934	0.4987	0.4190	0.3521	0.2959	0.2487
20%	C	1.2000	1.4400	1.7280	2.0736	2.4883	2.9860	3.5832	4.2998
	D	0.8333	0.6944	0.5787	0.4823	0.4019	0.3349	0.2791	0.2326
21%	C	1.2100	1.4641	1.7716	2.1436	2.5937	3.1384	3.7975	4.5950
	D	0.8264	0.6830	0.5645	0.4665	0.3855	0.3186	0.2633	0.2176
22%	C	1.2200	1.4884	1.8158	2.2153	2.7027	3.2973	4.0227	4.9077
	D	0.8197	0.6719	0.5507	0.4514	0.3700	0.3033	0.2486	0.2038
23%	C	1.2300	1.5129	1.8609	2.2889	2.8153	3.4628	4.2593	5.2389
	D	0.8130	0.6610	0.5374	0.4369	0.3552	0.2888	0.2348	0.1909
24%	C	1.2400	1.5376	1.9066	2.3642	2.9316	3.6352	4.5077	5.5895
	D	0.8065	0.6504	0.5245	0.4230	0.3411	0.2751	0.2218	0.1789
25%	C	1.2500	1.5625	1.9531	2.4414	3.0518	3.8147	4.7684	5.9605
	D	0.8000	0.6400	0.5120	0.4096	0.3277	0.2621	0.2097	0.1678

Range: n = 9 to 16, 1% to 12%

[*C* = multiplier for compounding; *D* = multiplier for discounting]

		n=9	*n*=10	*n*=11	*n*=12	*n*=13	*n*=14	*n*=15	*n*=16
1%	C	1.0937	1.1046	1.1157	1.1268	1.1381	1.1495	1.1610	1.1726
	D	0.9143	0.9053	0.8963	0.8874	0.8787	0.8700	0.8613	0.8528
2%	C	1.1951	1.2190	1.2434	1.2682	1.2936	1.3195	1.3459	1.3728
	D	0.8368	0.8203	0.8043	0.7885	0.7730	0.7579	0.7430	0.7284
3%	C	1.3048	1.3439	1.3842	1.4258	1.4685	1.5126	1.5580	1.6047
	D	0.7664	0.7441	0.7224	0.7014	0.6810	0.6611	0.6419	0.6232
4%	C	1.4233	1.4802	1.5395	1.6010	1.6651	1.7317	1.8009	1.8730
	D	0.7026	0.6756	0.6496	0.6246	0.6006	0.5775	0.5553	0.5339
5%	C	1.5513	1.6289	1.7103	1.7959	1.8856	1.9799	2.0789	2.1829
	D	0.6446	0.6139	0.5847	0.5568	0.5303	0.5051	0.4810	0.4581
6%	C	1.6895	1.7908	1.8983	2.0122	2.1329	2.2609	2.3966	2.5404
	D	0.5919	0.5584	0.5268	0.4970	0.4688	0.4423	0.4173	0.3936
7%	C	1.8385	1.9672	2.1049	2.2522	2.4098	2.5785	2.7590	2.9522
	D	0.5439	0.5083	0.4751	0.4440	0.4150	0.3878	0.3624	0.3387
8%	C	1.9990	2.1589	2.3316	2.5182	2.7196	2.9372	3.1722	3.4259
	D	0.5002	0.4632	0.4289	0.3971	0.3677	0.3405	0.3152	0.2919
9%	C	2.1719	2.3674	2.5804	2.8127	3.0658	3.3417	3.6425	3.9703
	D	0.4604	0.4224	0.3875	0.3555	0.3262	0.2992	0.2745	0.2519
10%	C	2.3579	2.5937	2.8531	3.1384	3.4523	3.7975	4.1772	4.5950
	D	0.4241	0.3855	0.3505	0.3186	0.2897	0.2633	0.2394	0.2176
11%	C	2.5580	2.8394	3.1518	3.4985	3.8833	4.3104	4.7846	5.3109
	D	0.3909	0.3522	0.3173	0.2858	0.2575	0.2320	0.2090	0.1883
12%	C	2.7731	3.1058	3.4785	3.8960	4.3635	4.8871	5.4736	6.1304
	D	0.3606	0.3220	0.2875	0.2567	0.2292	0.2046	0.1827	0.1631

Range: n = 9 to 16, 13% to 25%

[*C* = multiplier for compounding; *D* = multiplier for discounting]

		n=9	*n*=10	*n*=11	*n*=12	*n*=13	*n*=14	*n*=15	*n*=16
13%	C	3.0040	3.3946	3.8359	4.3345	4.8980	5.5348	6.2543	7.0673
	D	0.3329	0.2946	0.2607	0.2307	0.2042	0.1807	0.1599	0.1415
14%	C	3.2519	3.7072	4.2262	4.8179	5.4924	6.2613	7.1379	8.1372
	D	0.3075	0.2697	0.2366	0.2076	0.1821	0.1597	0.1401	0.1229
15%	C	3.5179	4.0456	4.6524	5.3503	6.1528	7.0757	8.1371	9.3576
	D	0.2843	0.2472	0.2149	0.1869	0.1625	0.1413	0.1229	0.1069
16%	C	3.8030	4.4114	5.1173	5.9360	6.8858	7.9875	9.2655	10.7480
	D	0.2630	0.2267	0.1954	0.1685	0.1452	0.1252	0.1079	0.0930
17%	C	4.1084	4.8068	5.6240	6.5801	7.6987	9.0075	10.5387	12.3303
	D	0.2434	0.2080	0.1778	0.1520	0.1299	0.1110	0.0949	0.0811
18%	C	4.4355	5.2338	6.1759	7.2876	8.5994	10.1472	11.9737	14.1290
	D	0.2255	0.1911	0.1619	0.1372	0.1163	0.0985	0.0835	0.0708
19%	C	4.7854	5.6947	6.7767	8.0642	9.5964	11.4198	13.5895	16.1715
	D	0.2090	0.1756	0.1476	0.1240	0.1042	0.0876	0.0736	0.0618
20%	C	5.1598	6.1917	7.4301	8.9161	10.6993	12.8392	15.4070	18.484
	D	0.1938	0.1615	0.1346	0.1122	0.0935	0.0779	0.0649	0.0541
21%	C	5.5599	6.7275	8.1403	9.8497	11.9182	14.4210	17.4494	21.1138
	D	0.1799	0.1486	0.1228	0.1015	0.0839	0.0693	0.0573	0.0471
22%	C	5.9874	7.3046	8.9117	10.8722	13.2641	16.1822	19.7423	24.0856
	D	0.1670	0.1369	0.1122	0.0920	0.0754	0.0618	0.0507	0.0415
23%	C	6.4439	7.9259	9.7489	11.9912	14.7491	18.1414	22.3140	27.4462
	D	0.1552	0.1262	0.1026	0.0834	0.0678	0.0551	0.0448	0.0364
24%	C	6.9310	8.5944	10.6571	13.2148	16.3863	20.3191	25.1956	31.2426
	D	0.1443	0.1164	0.0938	0.0757	0.0610	0.0492	0.0397	0.0320
25%	C	7.4506	9.3132	11.6415	14.5519	18.1899	22.7374	28.4217	35.5271
	D	0.1342	0.1074	0.0859	0.0687	0.0550	0.0440	0.0352	0.0281

2 Random sampling numbers

33865	04131	78302	22688	79034	01358	61724	98286	97086	21376
09356	09387	52825	93134	21731	93956	85324	68767	49490	11449
98243	37636	64825	43091	24906	13545	90172	31265	81457	93108
99052	61857	33938	86339	63531	77146	33252	81388	28302	18960
00713	24413	36920	03841	48047	04207	50930	84723	07400	81109

34819	80011	17751	03275	92511	70071	08183	72805	94618	46084
20611	34975	96712	32402	90182	94070	94711	94233	06619	34162
64972	86061	04685	53042	82685	45992	19829	45265	85589	83440
15857	73681	24790	20515	01232	25302	30785	95288	79341	54313
80276	67053	99022	36888	58643	96111	77292	03441	52856	95035

30548	51156	63914	64139	14596	35541	70324	20789	29139	66973
53530	79354	75099	89593	36449	66618	32346	37526	20084	52492
77012	18480	61852	82765	29602	10032	78925	71953	21661	95254
04304	40763	24847	07724	99223	77838	09547	47714	13302	17121
76953	39588	90708	67618	45671	19671	92674	22841	84231	59446

34479	85938	26363	12025	70315	58971	28991	35990	23542	74794
28421	16347	66638	25578	70404	67367	14730	37662	64669	16752
58160	17725	97075	99789	24304	63100	22123	83692	92997	58699
96701	73743	82979	69917	34993	36495	47023	48869	50611	61534
55600	61672	99136	73925	30250	12533	46280	03865	88049	13080

55850	38966	46303	37073	42347	36157	44357	52065	66913	06284
47089	83871	51231	32522	41543	22675	89316	38451	78694	01767
26035	86173	11115	22083	12083	43374	66542	23518	05372	33892
74920	35946	21149	70861	13235	02729	57485	23895	80607	11299
44498	00498	31354	39787	65919	61889	17690	10176	94138	95650

80045	71846	17840	23670	77769	84062	52850	20241	06073	20083
15828	95852	12124	95053	09924	91562	09419	27747	84732	81927
04100	75759	37926	70040	80884	48939	65228	60075	45056	56399
69257	48373	58911	78549	63693	43727	81058	53301	85945	54890
33915	26034	08166	59242	03881	88690	92298	48628	02698	94249

83497	62761	68609	85811	40695	08342	67386	63470	85643	68568
46466	15977	69989	90106	01432	59700	13163	56521	96687	41390
03573	87778	27696	35147	54639	20489	03688	72254	28402	98954
02046	44774	31500	30232	27434	14925	65901	34521	94104	54935
68736	12912	02579	34719	09568	21571	91111	81307	97866	76483

3 Exponential tables. Values of e^{-m}

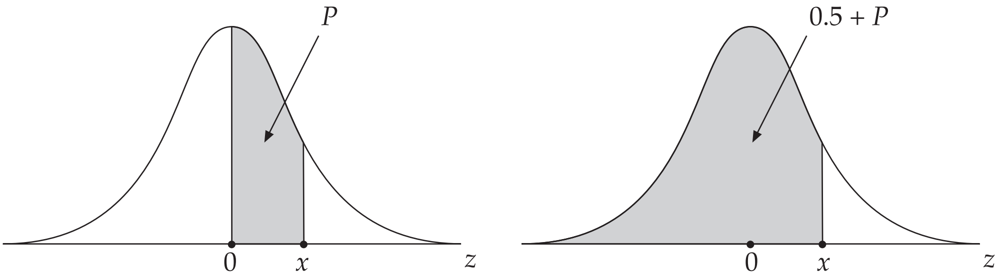
Range: $m = 0$ to 2.4

m	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
0.1	0.9048	0.8958	0.8869	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
0.2	0.8187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
0.3	0.7408	0.7334	0.7261	0.7189	0.7118	0.7047	0.6977	0.6907	0.6839	0.6771
0.4	0.6703	0.6637	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6126
0.5	0.6065	0.6005	0.5945	0.5886	0.5827	0.5769	0.5712	0.5655	0.5599	0.5543
0.6	0.5488	0.5434	0.5379	0.5326	0.5273	0.5220	0.5169	0.5117	0.5066	0.5016
0.7	0.4966	0.4916	0.4868	0.4819	0.4771	0.4724	0.4677	0.4630	0.4584	0.4538
0.8	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107
0.9	0.4066	0.4025	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
1.0	0.3679	0.3642	0.3606	0.3570	0.3535	0.3499	0.3465	0.3430	0.3396	0.3362
1.1	0.3329	0.3296	0.3263	0.3230	0.3198	0.3166	0.3135	0.3104	0.3073	0.3042
1.2	0.3012	0.2982	0.2952	0.2923	0.2894	0.2865	0.2837	0.2808	0.2780	0.2753
1.3	0.2725	0.2698	0.2671	0.2645	0.2618	0.2592	0.2567	0.2541	0.2516	0.2491
1.4	0.2466	0.2441	0.2417	0.2393	0.2369	0.2346	0.2322	0.2299	0.2276	0.2254
1.5	0.2231	0.2209	0.2187	0.2165	0.2144	0.2122	0.2101	0.2080	0.2060	0.2039
1.6	0.2019	0.1999	0.1979	0.1959	0.1940	0.1920	0.1901	0.1882	0.1864	0.1845
1.7	0.1827	0.1809	0.1791	0.1773	0.1755	0.1738	0.1720	0.1703	0.1686	0.1670
1.8	0.1653	0.1637	0.1620	0.1604	0.1588	0.1572	0.1557	0.1541	0.1526	0.1511
1.9	0.1496	0.1481	0.1466	0.1451	0.1437	0.1423	0.1409	0.1395	0.1381	0.1367
2.0	0.1353	0.1340	0.1327	0.1313	0.1300	0.1287	0.1275	0.1262	0.1249	0.1237
2.1	0.1225	0.1212	0.1200	0.1188	0.1177	0.1165	0.1153	0.1142	0.1130	0.1119
2.2	0.1108	0.1097	0.1086	0.1075	0.1065	0.1054	0.1044	0.1033	0.1023	0.1013
2.3	0.1003	0.0993	0.0983	0.0973	0.0963	0.0954	0.0944	0.0935	0.0926	0.0916
2.4	0.0907	0.0898	0.0889	0.0880	0.0872	0.0863	0.0854	0.0846	0.0837	0.0829

Range: m = 2.5 to 5.0

m	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.5	0.0821	0.0813	0.0805	0.0797	0.0789	0.0781	0.0773	0.0765	0.0758	0.0750
2.6	0.0743	0.0735	0.0728	0.0721	0.0714	0.0707	0.0699	0.0693	0.0686	0.0679
2.7	0.0672	0.0665	0.0659	0.0652	0.0646	0.0639	0.0633	0.0627	0.0620	0.0614
2.8	0.0608	0.0602	0.0596	0.0590	0.0584	0.0578	0.0573	0.0567	0.0561	0.0556
2.9	0.0550	0.0545	0.0539	0.0534	0.0529	0.0523	0.0518	0.0513	0.0508	0.0503
3.0	0.0498	0.0493	0.0488	0.0483	0.0478	0.0474	0.0469	0.0464	0.0460	0.0455
3.1	0.0450	0.0446	0.0442	0.0437	0.0433	0.0429	0.0424	0.0420	0.0416	0.0412
3.2	0.0408	0.0404	0.0400	0.0396	0.0392	0.0388	0.0384	0.0380	0.0376	0.0373
3.3	0.0369	0.0365	0.0362	0.0358	0.0354	0.0351	0.0347	0.0344	0.0340	0.0337
3.4	0.0334	0.0330	0.0327	0.0324	0.0321	0.0317	0.0314	0.0311	0.0308	0.0305
3.5	0.0302	0.0299	0.0296	0.0293	0.0290	0.0287	0.0284	0.0282	0.0279	0.0276
3.6	0.0273	0.0271	0.0268	0.0265	0.0263	0.0260	0.0257	0.0255	0.0252	0.0250
3.7	0.0247	0.0245	0.0242	0.0240	0.0238	0.0235	0.0233	0.0231	0.0228	0.0226
3.8	0.0224	0.0221	0.0219	0.0217	0.0215	0.0213	0.0211	0.0209	0.0207	0.0204
3.9	0.0202	0.0200	0.0198	0.0196	0.0194	0.0193	0.0191	0.0189	0.0187	0.0185
4.0	0.0183	0.0181	0.0180	0.0178	0.0176	0.0174	0.0172	0.0171	0.0169	0.0167
4.1	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
4.2	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
4.3	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
4.4	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
4.5	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
4.6	0.0101	0.0100	0.0099	0.0098	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092
4.7	0.0091	0.0090	0.0089	0.0088	0.0087	0.0087	0.0086	0.0085	0.0084	0.0083
4.8	0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0078	0.0077	0.0076	0.0075
4.9	0.0074	0.0074	0.0073	0.0072	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068
5.0	0.0067	0.0067	0.0066	0.0065	0.0065	0.0064	0.0063	0.0063	0.0062	0.0061

4 Standard Normal distribution tables



The table following gives the probability (P) that a Standard Normal variable lies between 0 and x.

This is equivalent to the shaded area in the left-hand figure.

To obtain the probability shown in the shaded area in the right-hand figure, 0.5 needs to be added to P as shown.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986

	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
	0.4987	0.4990	0.4993	0.4995	0.4997	0.4998	0.4998	0.4999	0.4999	0.5000

Index

Note that the references below refer to chapters and sections. Thus 20:11 refers to chapter 20, section 11.

10 to 90 percentile range 11:18

Absolute error 3:9
Accrued amount 21:7
Accrued amount of invested annuity 23:6
Activities, network 38:3
Activity list 38:3
Activity, dummy 38:5
Actual Percentage Rate (APR) 21:16
Addition and subtraction of matrices 36:6
Addition rule of probability 30:18
Additive model for time series 15:7
Amortization annuity 23:11
Amortization schedule 23:13
Amortized debt 23:11
Amount, accrued 21:7
Amount, principal 21:7
Annual Abstract of Statistics 2:23
Annuity 23:2
Annuity, amortization 23:11
Annuity, certain 23:3
Annuity, contingent 23:3
Annuity, due 23:3
Annuity, NPV 23:9
Annuity, ordinary 23:3
Annuity, perpetual 23:3
Approximate number 3:9
Arbitrary constant 27:13
Area graphs 5:19
Arithmetic mean 6:2
Arithmetic progression 21:3
Asymptote 24:12
Average Earnings Indices 20:11
Average inventory level 37:8
Axis-intersection ratio method 24:4, 35:8

Back-to-back bar chart 5:9
Bar chart, back-to-back 5:9
Bar chart, component 5:14
Bar chart, loss or gain 5:9
Bar chart, multiple 5:14
Bar chart, percentage 5:14
Bar chart, simple 5:8
Base changing, index numbers 18:9
Basic inventory model 37:14
Basic inventory model cost equation 37:15
Bayes theorem 31:13
Bias 2:5
Biased errors 3:17
Biased rounding 3:16
Binomial distribution 33:2
Binomial probability formula 33:4
Binomial situation 33:2
Binomial, mean and variance 33:7
Blue Book 2:23
Boundaries, class 4:9
Break-even points 26:11
British Business 2:23

Calculator, use of accumulating memory 6:13
Calculator, use with compound interest 21:13
Calculator, use with regression line 13:12
Calculator, use with standard deviation 10:8
Calculus 27:2
Capital investment 22:8
Carrying costs 37:3
Cash flow, discounted 22:9
Causal relationship 14:21
Census 2:3
Census of Distribution 2:3
Census of Production 2:3
Centred moving average 16:11
Certain annuity 23:3
Chain base relatives 18:7
Chain linking 18:16
Charts, Gantt 5:22
Charts, strata 5:19
Circular diagrams 5:11
Class boundaries 4:9
Class limits 4:9
Class mid-points 4:9
Class widths 4:9
Classes, open-ended 4:8
Classification of data 3:2
Cluster sampling 2:15
Coefficient of correlation 14:4
Coefficient of determination 14:13
Coefficient of dispersion, quartile 11:15
Coefficient of variation 10:14
Combinations 32:2
Commodity, real value of 18:12
Common difference 21:3
Common ratio 21:5
Comparative histograms 4:15
Comparisons of mean, median and mode 8:10
Compensating errors 3:17
Complement of a set 29:4
Complementary rule of probability 30:17
Component bar charts 5:14
Component time series 5:3
Components of composite index number 19.2
Components of time series 15:8
Composite index number 19:2
Compound experiment 30:22
Compound interest 21:8
Compound interest tables 21:13
Conditional probability 31:6
Conditional probability, definition 31:7
Confidence limits 34:15
Confidence limits for mean 34:16
Confidence limits for proportion 34:17
Constant, arbitrary 27:13
Contingent annuity 23:3
Continuous data 3:4
Control, inventory 37:4
Correlation 14:2

Correlation coefficient 14:4
Correlation, negative 14:8
Correlation, positive 14:7
Correlation, spurious 14:21
Cost equation, basic inventory model 37:15
Cost function 28:6
Cost of capital 22:22
Cost, marginal function 28:13
Cost, set-up 37:19
Costs, holding or carrying 37:3
Costs, ordering or replenishment 37:3
Costs, stockout 37:3
Critical path 38:8
Cubic equations, general form 26:12
Cubic equations, graphical solution 26:13
Cubic functions and equations 24:9
Cumulative frequency curve 4:20
Cumulative frequency distribution 4:19
Cumulative frequency polygon 4:20
Cumulative line diagrams 5:19
Curve, cumulative frequency 4:20
Curve, frequency 4:16
Curve, Lorenz 4:23
Curvilinear regression 13:20
Cycle of inventory 37:8
Cycle of time series 15:4

Data array 4:3
Data classification 3:2
Data collection 2:19
Data, continuous 3:4
Data, deseasonalised 17:16
Data, discrete 3:3
Data, primary 2:2
Data, raw 4:2
Data, secondary 2:2
Debt repayment, sinking fund 23:16
Deciles 11:18
Deflation of time series 18:13
Demand (price) function 28:9
Dependent variable 13:13
Depreciation fund 23:19
Depreciation schedule 23:19
Depreciation, reducing balance 21:20
Depreciation, sinking fund 23:19
Depreciation, straight line 21:20
Derivative of a function 27:4
Derivative, second 27:8
Deseasonalised data 17:16
Design of questionnaires 2:20
Determinant of matrix 36:18
Determination, coefficient of 14:13
Deviation, mean 9:7
Deviation, quartile 11:6
Deviation, standard 10:1
Diagram, line 5:12
Diagrams, cumulative line 5:19
Diagrams, purpose of 5:4
Diagrams, Venn 29:5
Differentiation 27:2
Discount factor 22:4
Discount rate 22:2
Discounted cash flow 22:9
Discounting tables 22:4
Discrete data 3:3
Disjoint sets 29:12
Dispersion, measures of 6:0
Distribution, binomial 33:2
Distribution, census of 2:3
Distribution, frequency 4:4
Distribution, poisson 33:9
Distributions, less than 4:19
Distributions, more than 4:19
Divided circles 5:11
Due annuity 23:3
Dummy activity 38:5

Economic batch quantity (EBQ) 37:19
Economic ordering quantity (EOQ) 37:16
Economic Trends 2:23
Effective interest rate 21:16
Elements of a set 29:2
Empirical probability 30:11
Empirical probability, definition 30:14
Employment Gazette 2:23
Enumeration notation, 2-set 29:18
Enumeration notation, 3-set 29:20
Enumeration of sets 29:13
Enumeration problem, general 29:16
Equality, set 29:4
Equally likely outcomes 30:4
Equation 24:2
Equation, cubic 24:9
Equation, linear form 24:3
Equation, quadratic form 24:3
Equations involving ratios 25:19
Equations, cubic 26:12
Equations, linear 25:2
Equations, quadratic 26:2
Equations, simultaneous linear 25:2
Error avoidance with percentages 3:19
Error rules 3:13
Error, absolute 3:9
Error, relative 3:9, 3:18
Errors in expressions 3:12
Errors, biased 3:17
Errors, compensating 3:17
Errors, rounding 3:11
Errors, unpredictable 3:7
Event, statistical 30:5
Events, independent 30:9
Events, mutually exclusive 30:7
Events, network 38:3
Events, split 31:11
Expectation 31:2
Expected value 31:2
Experiment, compound 30:22
Explained variation 14:12
Exploding sectors 5:11
Extrapolation 13:17

Factor, discount 22:4
Factor, present value 22:4
Factorials 32:4
Factorisation for quadratic equations 26:7
Fair rounding 3:6, 3:16
Family Expenditure Survey 20:5
Feasible region 35:4
Financial Statistics 2:23
First derivative 27:4
Fixed base relatives 18:7
Fixed costs 28:6
Float 38:12

Forecasting with time series 17:9
Formula, amortized debt payment 23:11
Formula, APR calculation 21:18
Formula, Bayes theorem 31:13
Formula, binomial mean and variance 33:7
Formula, binomial probability 33:4
Formula, coefficient of determination 14:13
Formula, coefficient of variation 10:14
Formula, cost function 28:6
Formula, demand function 28:9
Formula, derivative of a function 27:4
Formula, EBQ 37:19
Formula, EOQ 37:16
Formula, expected value 31:3
Formula, factorial 32:4
Formula, geometric mean 8:14
Formula, harmonic mean 8:17
Formula, integration of a function 27:13
Formula, internal rate of return (IRR) 22:16
Formula, Laspeyres index 19:11
Formula, mean deviation 9:8
Formula, mean for distribution 6:11
Formula, mean for set 6:6
Formula, median by interpolation 7:9
Formula, mode by interpolation 8:4
Formula, number of combinations 32:5
Formula, number of permutations 32:6
Formula, Paasche index 19:12
Formula, Pearson's measure of skewness 10:16
Formula, poisson probability 33:11
Formula, present value 22:3
Formula, present value of debt 22:6
Formula, price relative 18:6
Formula, profit function 28:3
Formula, proportional pie charts 5:18
Formula, quantiles by interpolation 11:13
Formula, quantity relative 18:6
Formula, quartile coefficient of dispersion 11:14
Formula, quartile deviation 11:6
Formula, quartile measure of skewness 11:16
Formula, rank correlation 14:15
Formula, real value index 18:13
Formula, reducing balance depreciation 21:23
Formula, revenue function 28:8
Formula, seasonal adjustment 17:7
Formula, solution of quadratic equations 26:5
Formula, split event rule 31:11
Formula, standard deviation, distribution 10:10
Formula, time series forecasting 17:11
Formula, weighted aggregate index 19:7
Formula, weighted average of relatives 19:6
Formula, Z-score 34:5
Formulae, arithmetic progression 21:3
Formulae, geometric progression 21:5
Formulae, interest amount accrued 21:9
Free float 38:12
Frequency 4:4
Frequency curve 4:16
Frequency distribution 4:4
Frequency distribution, cumulative 4:19
Frequency distribution, formation of 4:11
Frequency distribution, grouped 4:7
Frequency distribution, relative 4:10
Frequency distribution, simple 4:4
Frequency distributions, non-numeric 5:2
Frequency polygon 4:16
Function 24:2
Function derivative 27:4
Function, cost 28:6
Function, linear 12:2
Function, marginal 28:13
Function, objective 35:8
Function, profit 28:3
Function, revenue 28:8
Functions, ratio of two linear 24:12
Functions, simple 27:4
Fund, depreciation 23:19
Fund, sinking 23:15

Gantt charts 5:22
Geometric mean 8:13
Geometric mean calculations 8:16
Geometric progression 21:5
Gradient of straight line 12:2
Gradient, determining 12:6
Gradual replenishment model 37:18
Graph of inventory 37:6
Graph of time series 15:10
Graph, semi-logarithmic 5:23
Graphical estimation of IRR 22:15
Graphical estimation of median 7:11
Graphical estimation of mode 8:7
Graphical estimation of quartiles 11:11
Graphical solution, cubic equations 26:13
Graphical solution, quadratic equations 26:8
Graphical solution, simultaneous equations 25:13
Graphs of quadratic functions 24:6
Graphs, area 5:19
Grouped frequency distribution 4:7

Harmonic mean 8:17
Heterogeneous population 2:11
Histogram 4:13
Histograms for unequal classes 4:28
Histograms, comparative 4:15
Historigram 15:10
Historigrams 5:13
Holding costs 37:3
Homogeneous population 2:13
Household Expenditure Survey 20:5

Ideograms 5:7

Independent events 30:9
Independent float 38:12
Independent variable 13:13
Index number 18:2
Index number base changing 18:9
Index number construction considerations 19:15
Index number limitations 19:17
Index number notation 18:4
Index number uses 19:16
Index number, composite 19:2
Index number, points increase 18:16
Index Numbers of Producer Prices 20:10
Index of Industrial Production 20:12
Index of Output of Production Industries 20:12
Index of Retail Prices 20:2
Index of Retail Sales 20:14
Index relative 18:5
Index, Laspeyres 19:11
Index, Paasche 19:12
Index, real value 18:14
Index, Tax and Price 20:8
Index, Wholesale Price 20:10
Indices of Average Earnings 20:11
Industrial Production Index 20:12
Inequalities 35:2
Inequalities, sign technique 35:3
Inspection method, for regression line 13:4
Integration 27:2, 27:13
Interest rate 21:7
Interest rate, effective 21:16
Interest rate, nominal 21:16
Interest, compound 21:8
Interest, simple 21:8
Internal rate of return (IRR) 22:14
Interpolation 13:17
Interpolation formula for median 7:7
Interpolation formula for mode 8:4
Interpolation formula for quantiles 11:13
Interquartile range 11:7
Intersection of sets 29:9
Interview, street 2:19
Interview, telephone 2:19
Interviewer bias 2:5
Inventory 37:2
Inventory control 37:4
Inventory control systems 37:10
Inventory cycle 37:8
Inventory graph 37:6
Inventory models 37:13
Inventory system, periodic review 37:10
Inventory system, reorder level 37:10
Inventory, average level 37:8
Inverse of matrix 36:18
Invested annuity, accrued amount 23:6
Investigation, statistical 1:7
Investment schedule 21:10
IRR, graphical estimation 22:15

Laspeyres index 19:11
Lead time 37:5
Least squares regression 13:9
Least squares regression formula 13:10
Least squares, time series trend 16:7
Less than distributions 4:19
Limits of probability 30:17
Limits, class 4:9
Line diagram 5:13
Line diagrams, cumulative 5:19
Line diagrams, multiple 5:13
Line of equal distribution 4:24
Linear equation form 24:3
Linear equation manipulation 25:8
Linear equations 25:2
Linear equations, algebraic solution 25:4
Linear equations, graphical solution 25:5
Linear function 12:2
Linear inequalities 35:3
Linear programming 35:6
Location, measures of 6:0
Long-term cycles 15:8
Lorenz curve 4:23
Loss or gain bar chart 5:9

Management mathematics 1:5
Manipulation of mean formula 6:16
Marginal functions 28:13
Mathematical limits 4:9
Matrix 36:2
Matrix multiplication, conditions 36:12
Matrix multiplication, with matrix 36:11
Matrix multiplication, with number 36:9
Matrix, addition and subtraction 36:6
Matrix, determinant 36:18
Matrix, inverse 36:19
Matrix, transition 36:23
Matrix, unit 36:18
Maximum (local) point 24:6
Maximum (stock) level 37:5
Maximum error 3:8
Maximum profit point 28:3
Mean deviation 9:7
Mean deviation characteristics 9:11
Mean deviation formula 9:8
Mean for a set 6:6
Mean formula manipulation 6:16
Mean of binomial 33:7
Mean point, regression 13:16
Mean validation 6:13
Mean, arithmetic 6:6
Mean, geometric 8:13
Mean, grouped frequency distribution 6:10
Mean, harmonic 8:17
Mean, simple frequency distribution 6:8
Mean, test of significance 34:19
Mean, weighted 6:17
Measures of dispersion 6:0
Measures of location 6:0
Measures of skewness 6:0
Median by interpolation 7:7
Median characteristics 7:16
Median for a set 7:2
Median for simple frequency distribution 7:6
Median, comparison of techniques 7:14
Members 1:7

Members of a set 29:2
Minimum (local) point 24:6
Minimum (stock) level 37:11
Mode 8:2
Mode characteristics 8:9
Mode, by interpolation 8:4
Models for time series 15:5
Monthly Digest of Statistics 2:23
More than distributions 4:19
Moving average centring 16:11
Moving averages 16:9
Moving totals 5:20
Multi-stage sampling 2:14
Multiple bar charts 5:14
Multiple line diagrams 5:13
Multiple pie charts 5:17
Multiple regression 13:20
Multiple time series 5:3
Multiplication rule of probability 30:20
Multiplication rule, general 31:10
Multiplicative model for time series 15:7
Mutually exclusive events 30:7

Negative correlation 14:8
Net present value (NPV) 22:10
Network planning 38:1
Nominal interest rate 21:16
Non-causal relationship 13:14
Non-numeric frequency distributions 5:2
Non-random sampling 2:7
Normal approximation to binomial 34:12
Normal distribution 34:2
Normal distribution probability 34:4
Notation, index number 18:4
Notation, sigma (summation) 6:5
NPV of annuity 23:9
Number of a set 29:4

Objective function 35:8
Ogive 4:20
One-sided errors 3:17
Open-ended classes 4:8
Optimum solution 35:8
Ordering costs 37:3
Ordinary annuity 23:3
Outcome set 30:4
Outcomes 30:4
Outcomes, equally likely 30:4

Paasche index 19:12
Parabola 24:6
Path, critical 38:8
Pearson's measure of skewness 10:16
Percentage bar charts 5:14
Percentiles 11:18
Period of moving average 16:9
Periodic review inventory system 37:12
Permutations 32:2
Perpetual annuity 23:3
Personal interview 2:19
Pictograms 5:6
Pie charts 5:10
Pie charts, multiple 5:17
Pie charts, proportional 5:18
Pilot survey 1:8
Pink Book 2:23
Points increase (index number) 18:16
Poisson approximation to the binomial 33:15
Poisson distribution 33:9
Poisson probability formula 33:11
Poisson process 33:9
Poisson situation 33:9
Poisson, interval adjustment 33:13
Polygon, cumulative frequency 4:20
Polygon, frequency 4:16
Polynomial 24:11
Population 1:8
Population census 2:3
Population, heterogeneous 2:11
Population, homogeneous 2:13
Positive correlation 14:7
Postal questionnaire 2:19
Precedence table 38:3
Precision 2:17
Predictable errors 3:7
Present value 22:2
Present value factor 22:4
Present value, interest bearing debt 22:6
Price (demand) function 28:9
Price relative 18:5
Primary data 2:2
Principal amount 21:7
Probability 1:6, 30:2
Probability formula, binomial 33:4
Probability formula, poisson 33:11
Probability limits 30:17
Probability table 31:4
Probability, addition rule 30:18
Probability, complementary rule of 30:17
Probability, conditional 31:6
Probability, empirical 30:11
Probability, multiplication rule 30:20
Probability, normal distribution 34:4
Probability, theoretical 30:11
Probability, total rule of 30:17
Process, poisson 33:9
Producer Prices, Index Numbers 20:10
Product moment correlation coefficient 14:6
Product moment correlation formula 14:5
Production Industries, Index of Output 20:12
Production run model 37:18
Production, census of 2:3
Profit function 28:3
Profit point, maximum 28:3
Programming, linear 35:6
Progression, arithmetic 21:3
Progression, geometric 21:5
Projecting a time series trend 17:12
Projection of time series 17:9
Proportional pie charts 5:18
Purchasing Power of the Pound (Index) 20:7

Quadratic equation form 24:3
Quadratic equations, factorisation 26:7
Quadratic equations, general form 26:2
Quadratic equations, graphical solution 26:8
Quadratic function graphs 24:6
Quantile 11:2

Quantiles, interpolation formula 11:13
Quantity relative 18:5
Quartile coefficient of dispersion 11:15
Quartile deviation 11:7
Quartile measure of skewness 11:16
Quartiles 11:3
Quartiles for frequency distribution 11:9
Quartiles, graphical estimation 11:11
Quasi-random sampling 2:7
Questionnaire design 2:20
Quote sampling 2:16

Random factors (variation) 15:8
Random sampling 2:8
Random sampling numbers 2:8
Range 9:4
Range characteristics 9:6
Range, 10 to 90 percentile 11:18
Range, semi-interquartile 11:6
Rank correlation coefficient 14:15
Rank correlation, uses 14:16
Rate of change 27:7
Rate of interest 21:7
Rate of return, internal 22:14
Ratios in equations 25:19
Raw data 4:2
Real value index 18:14
Real value of commodity 18:12
Recording bias 2:5
Reducing balance depreciation 21:20
Region, feasible 35:4
Regional Trends 2:23
Regression 13:2
Regression line, inspection 13:4
Regression line, least squares 13:9
Regression line, semi-averages 13:6
Regression line, x on y 13:6
Regression line, y on x 13:6
Regression mean point 13:16
Regression, curvilinear 13:20
Regression, multiple 13:20
Relative error 3:9, 3:18
Relative frequency distribution 4:10
Relative frequency probability 30:11
Relative, index 18:5
Relative, price 18:5
Relative, quantity 18:5
Relatives, chain base 18:7
Relatives, fixed base 18:7
Reorder level 37:5
Reorder level inventory system 37:11
Reorder quantity 37:5
Replenishment costs 37:3
Residual variation 15:8, 17:14
Retail Prices Index (RPI) 20:2
Retail Sales Index 20:14
Revenue function 28:8
Revenue, marginal function 28:13
Rounding conventions 3:6
Rounding errors 3:11
Rounding, biased 3:16
Rounding, fair 3:6, 3:16
Rules of errors 3:13
Run size 37:18
Run time 37:18

Safety stock 37:5
Sample 1:8, 2:4
Sample size 2:18
Sampling frame 2:6
Sampling, cluster 2:15
Sampling, multi-stage 2:14
Sampling, non-random 2:7
Sampling, quasi-random 2:7
Sampling, quota 2:16
Sampling, random 2:7
Sampling, simple random 2:7
Sampling, stratified random 2:12
Sampling, systematic 2:13
Schedule, amortization 23:13
Schedule, depreciation 23:19
Schedule, investment 21:10
Schedule, sinking fund 23:17
Seasonal adjustment 17:6
Seasonal variation 17:1
Second derivative 27:8
Secondary data 2:2
Secondary data sources 2:22
Secondary statistics 2:2
Sectors of pie chart 5:10
Selection bias 2:5
Semi-averages regression line 13:7
Semi-averages, time series trend 16:4
Semi-interquartile range 11:6
Semi-logarithmic graph 5:24
Set enumeration 29:13
Set equality 29:4
Set intersection 29:9
Set number 29:4
Set union 29:7
Set, complement 29:4
Set, outcome 30:4
Set, universal 29:4
Set-up Cost 37:19
Set-up costs 28:6
Sets 29:2
Sets, disjoint 29:12
Sigma notation 6:5
Sign technique for inequalities 35:3
Significance test for mean 34:19
Simple bar chart 5:8
Simple frequency distribution 4:4
Simple functions 27:3
Simple interest 21:8
Simple random sampling 2:9
Simultaneous equations, graphical solution 25:13
Simultaneous equations, matrix solution 36:20
Simultaneous linear equations 25:2
Sinking fund 23:15
Sinking fund debt repayment 23:16
Sinking fund depreciation 23:19
Sinking fund schedule 23:17
Size of sample 2:18
Skewness 9:13
Skewness, measures of 6:0
Skewness, Pearson's measure of 10:16
Skewness, quartile measure of 11:16
Social Trends 2:23
Solution, 2 x 2 simultaneous equations 25:9
Solution, 3 x 3 simultaneous equations 25:16

Solution, linear equations 25:3
Solution, optimum 35:8
Sources of secondary data 2:22
Spearman's rank correlation coefficient 14:15
Split event rule 31:11
Split events 31:11
Spurious correlation 14:21
Standard deviation 10:2
Standard deviation characteristics 10:13
Standard deviation for a set 10:3
Standard deviation, computational formula 10:6
Standard deviation, derivation 10:18
Standard deviation, frequency distribution 10:11
Standard deviation, set, formula 10:2
Standard error 34:16
Standard normal tables 34:6
States (of transition) 36:23
Statistical diagrams, purpose of 5:4
Statistical event 30:5
Statistical experiment 30:3
Statistical investigation 1:7
Statistical method 1:4
Statistics 1:4
Statistics, Annual Abstract 2:23
Statistics, secondary 2:2
Stock, maximum 37:5
Stock, safety 37:5
Stockout costs 37:3
Straight line 12:3
Straight line depreciation 21:20
Straight line, finding equation of 12:8
Strata charts 5:19
Stratification 2:11
Stratified random sampling 2:12
Street interview 2:19
Structure bias 2:5
Subjective probability 30:11
Subsets 29:4
Summation notation 6:5
Survey 1:7
Survey, Family Expenditure 20:5
Survey, Household Expenditure 20:5
Survey, pilot 1:8
Survey, trial 1:8
Systematic errors 3:17
Systematic sampling 2:13
Systems of inventory control 37:10

Table, probability 31:4
Tables, compound interest 21:13
Tables, discounting 22:4
Tables, standard normal 34:6
Tally chart 4:4
Tax and Price Index 20:8
Telephone interview 2:19
Terms of a progression 21:3
Test of significance for mean 34:19
Theoretical probability 30:11
Theoretical probability, definition 30:12
Time series 5:3, 15:2
Time series additive model 15:7
Time series analysis 15:1
Time series components 15:8
Time series cycle 15:4
Time series deflation 18:13
Time series forecasting 17:9
Time series graph 15:10
Time series multiplicative model 15:7
Time series of relatives 18:7
Time series, component 5:3
Time series, multiple 5:3
Total float 38:12
Total probability rule 30:17
Transition matrix 36:23
Transition states 36:23
Tree diagram 30:23
Trend for time series 16:1
Trend projection, time series 17:12
Trend values significance 16:2
Trial survey 1:8
Trials, binomial 33:2
Turning point 24:11
Turning points, identification of 27:10

Unexplained variation 14:12
Union of sets 29:7
Unit matrix 36:18
Universal set 29:4
Unpredictable errors 3:7

Validation of mean 6:13
Validation of standard deviation 10:12
Value, expected 31:2
Variable costs 28:6
Variance of binomial 33:7
Variation, coefficient of 10:14
Variation, explained 14:12
Variation, unexplained 14:12
Venn diagrams 29:5

Weighted aggregate index 19:7
Weighted average of relatives index 19:6
Weighted mean 6:17
Wholesale Price Index 20:10
Wording bias 2:5
Worth of money 22:22

X on y regression line 13:6

Y on x regression line 13:6
Y-intercept of straight line 12:2